Characteristics of Teachers' Conversations about Teaching Mean, Median, and Mode

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ABSTRACT

The study analyzed a conversation among a group of teachers responsible for teaching the concepts of mean, median, and mode. After reading an article describing some specific student difficulties in learning the concepts, teachers were asked to discuss how the teaching of the concepts could be improved. Several claims pertinent to improving teaching practice were offered. Claims focused on the appropriate age at which to introduce statistical concepts, the influence of the state-prescribed curriculum on teaching practice, content-specific teaching strategies, and content-independent teaching strategies. Teachers' claims were discussed in terms of points of departure and agreement with existing empirical research.

1. INTRODUCTION

During the past two decades, the attention given to statistics as part of school curricula has increased dramatically. Rather than being reserved for study at the tertiary level, statistics is now recommended as a component of students' earliest school experiences (Australian Education Council, 1994; DfEE, 1999; National Council of Teachers of Mathematics, 2000). Shaughnessy (2007) observed that the more prominent position given to statistics in many curriculum documents around the world has sparked unprecedented levels of research activity. The increased research activity was evidenced by the appearance of new venues for the dissemination of statistics education research, such as *Statistics Education Research Journal* and *Journal of Statistics Education*. In a relatively short period of time, statistics went from a rarely-addressed subject in mathematics classes to a subject fueling an emerging body of research that supplements and extends mathematics education research.

Within the emerging discipline of statistics education research, the statistical concepts of mean, median, and mode have been responsible for motivating myriad studies. One key finding is that students may understand procedures for determining the three statistical measures without deeply understanding their mathematical properties (Strauss & Bichler, 1988) or applications (Mokros & Russell, 1995; Watson & Moritz, 2000). The way algorithms are taught in school can interfere with students' ability to move beyond surface-level procedural understanding to appreciate the mean, median, and mode as measures of center or as tools for other forms of data analysis (Groth, 2005). Likewise, teachers themselves have shown difficulty in attaining this sort of conceptual understanding, even after taking formal courses in introductory statistics (Groth & Bergner, 2006). The persistence of difficulties with learning and teaching the foundational ideas of mean, median, and mode indicates a need for further research to unpack underlying causes.

Although the nature of individuals' knowledge of mean, median, and mode has received a great deal of attention in the literature, the manner in which interactions among teachers influence the teaching of the concepts has not. The discourse communities that teachers inhabit can exert a great deal of influence over the way subject matter is taught. At times, the influences are positive, as when teachers form collegial communities of practice (Lave & Wenger, 1991) with one another. These might become formal structures like lesson study groups (Lewis, 2002) or content study groups (Zevenbergen, 2004) in which teachers learn through challenging and extending one another's knowledge of pedagogy and content. Productive discourse among teachers may also remain at a more informal level and center upon discussions of curricular materials (Ma, 1999) and mathematics problems (Davis & Simmt, 2003).

It should be noted, however, that not all discourse among teachers helps support effective approaches to teaching. Teachers' discourse communities can exert unpredictable and sometimes counterproductive influences on teachers' perceptions and growth. Putnam and Borko (2000) argued, "patterns of classroom teaching and learning have historically been resistant to fundamental change, in part because schools have served as powerful discourse communities that enculturate participants (students, teachers, administrators) into traditional school activities and ways of thinking" (p. 8). Stigler and Hiebert (1999) expressed a similar sentiment in portraying mathematics teaching in the U.S. as a cultural system that is highly resistant to change. Efforts aimed at improving the quality of mathematics instruction have sometimes caused unintended dynamics among educators. The release of the NCTM (1989) Standards, for example, sparked the "Math Wars" because the document's authors saw it as an endorsement of a richer view of mathematics than what was provided by conventional curricula, but critics saw it as a retreat from rigorous mathematics (Schoen, Fey, Hirsch, & Coxford, 1999; Schoenfeld, 2004). Hence,

in order to understand why content is taught as it is, educators' discourse among one another should be taken into account.

2. PURPOSE OF THE STUDY

The purpose of the present study was to describe and apply a method for attending to teachers' conversations about teaching mean, median, and mode. Attending to teachers' discourse about teaching the three concepts and knowing some of the themes that pervade the discourse can help inform the substance of teacher education efforts and curricular recommendations for teaching statistics. The specific research questions used to guide the investigation were: (i) In conversations about teaching the mean, median, and mode, what kinds of claims do teachers make about how the teaching of the concepts can be improved? (ii) How do teachers support their claims?

3. METHOD

3.1 *Study design*

A case study approach (Stake, 2000) was taken to investigating discourse among teachers about teaching the statistical concepts of mean, median, and mode. The study was designed to be a micro-level analysis of a specific conversation rather than a macro-level study for making statistical generalizations about the types of discourse that may generally occur among teachers when discussing the concepts. Although statistical generalization was not a goal, the case was intended to contribute to theory (Yin, 2003) and to methods for research by carrying out an under-utilized approach to studying the causes of difficulties with teaching and learning mean, median, and mode.

3.2 *Participants and procedure*

The conversation analyzed in the present study occurred as part of a larger professional development program provided to schools by the author's university. Nine elementary and middle school teachers representing 5 different school districts (Table 1) participated in the discussion of an article entitled, *Mean and Median: Are They Really So Easy?* (Zawojewski & Shaughnessy, 2000). The article gave examples of tasks used on the National Assessment of Educational Progress (NAEP) to assess students' understanding of measures of central tendency. It also provided details about students' responses to the tasks. The student work samples and data presented in the article helped emphasize the idea that many students do not understand the relative advantages or meanings of the concepts of mean, median, and mode. Studying students' learning of difficult topics like these provides a means for teachers to develop knowledge of content and knowledge of students in tandem as they discuss artifacts associated with students' learning (Smith, 2001). Hence, the procedure used for the study addressed teacher education goals alongside the research goal of examining the nature of teachers' discourse.

<INSERT TABLE 1 HERE>

The article discussion took place in an asynchronous online discussion board environment because of its potential to: (i) Allow teachers extended time to reflect on one another's comments (Newell, Wilsman, Langenfeld, & McIntosh, 2002); (ii) Encourage contributions from those who may normally be silent in face-to-face settings (Groth, 2006a); and (iii) Spark productive debates about pedagogical ideas (Groth, 2007). The conversation lasted one week, and teachers were asked to make at least four posts to the online discussion board. An informal but explicitly stated goal was to make two posts early in the week and another two later on, in order to allow time for

individuals to read and reflect on comments made by others. Each teacher received

approximately $60 (US) for participating in online discussion in accord with these guidelines

(the actual amount varied slightly with the stipend for overtime rate allowed by each teacher's

school district). Teachers were told in advance they would receive the $60 payment, or a portion

thereof, if they did not meet all of the participation expectations. The payment was not related in

any way to the normal salaries they received from their schools. They were free to withdraw

from the conversation without penalty. Teachers were asked to pay special attention to students'

thinking as it was described in the article (Zawojewski & Shaughnessy, 2000), and then discuss

how what was learned about students' thinking could be used to help improve instruction.

3.3 *Data gathering and analysis*

The complete transcript of the discussion board conversation was retained for analysis.

Discussion board messages that contained claims about how teaching practices could be

improved became focal points for analysis. A matrix for sorting the qualitative data (Miles &

Huberman, 1994) was constructed that contained the claims made as well as the support teachers

provided for the claims. Support for the claims was conceived of in terms of Toulmin's (1969)

categories of data, warrant, and backing. Hence, when a message contained a claim about how

the teaching of statistics could be improved, the supporting data provided for the claim (e.g.,

personal experience with students, student achievement data from the article under

consideration) was placed in a corresponding cell in the matrix. Messages were also analyzed for

the presence of warrants, which help justify one's interpretation of data; and backing statements,

which provide further support for the warrant. When warrants and backing statements were

present, they were placed in the matrix cells alongside the claims they supported.

A second matrix was also constructed, in which similar types of claims were clustered together. The second matrix became the basis for writing a narrative of the teachers' discussion. Data analysis continued with the writing of the narrative (Glesne, 1999; Richardson & St. Pierre, 2005). As the narrative was written, the initial inferences drawn from the data analysis matrix were compared against the original data (LeCompte & Preissle, 1993), so that conclusions drawn early on in the analysis would be subject to possible falsification (Kirk & Miller, 1986). In addition, several examples from the data are included in the narrative to help the reader form his or her own conclusions about the accuracy of the treatment of the data and the viability and utility of the conclusions and implications drawn (Lincoln & Guba, 1985).

4. RESULTS

Table 2 shows the types of claims teachers made during the discussion, along with the number of teachers making each type of claim. Some claims focused on the age at which meanings for measures of center should be taught to students. Another category of claims focused on relationships between the existing state-prescribed curriculum and students' learning. Others described content-specific methods for teaching the concepts, like using box plots and meaningful data sets. Still other claims emphasized that content-independent methods needed more attention, such as vocabulary instruction and technology usage.

<INSERT TABLE 2 HERE>

4.1 *Should meanings for measures of center be taught to young students?*

One of the most pervasive claims on the discussion board was that meanings for measures of center should not be taught to young students (although an age cutoff point was never specified). Felicia cited her own frustrations with attempting to teach for meaning with her students as

evidence for the claim, stating, "It is to beat your head against a wall to have them interpret [mean, median, and mode]." Felicia also cited the data from the article used to catalyze the conversation as evidence for her claim, emphasizing the low success rates it documented for students on tasks that required deciding which measure of center to use in different situations. Heather and Joseph cited the personal frustrations they experienced in coming to understand mean, median, and mode as evidence they should not be taught early on in school. Heather at one point, for example, offered her experience from studying at home with a parent as evidence:

> I vividly remember struggling with that as a junior high and probably high school student… There were many tearful arguments I'd have with my Dad who never believed in memorizing algorithms, he insisted that I understand how and why they work so that I could reconstruct them when I needed to and could use the "best" one for the task.

Ingrid offered her own students' struggles as evidence that meaning for measures of center should not be studied by young students, stating, "When asked what are the measures of central tendency, they will reply with range as an answer (along with mode and mean usually). They are confused with measures of center and measures of variability."

The warrant for interpreting learners' failure in understanding mean, median, and mode as evidence to support the practice of waiting until students are older to teach for understanding was that younger students are not developmentally ready for the task. Felicia, Heather, and Joseph all felt that students were not innately ready to learn with understanding early on. Heather, for example, linked her initial difficulties in learning the concepts from her father to age differences, stating, "I think the trouble was that Dad saw the concepts from a 50-year-olds' chemical engineer perspective and that was beyond my 13-18 year-old abstract ability." Ingrid

hypothesized that environmental, rather than innate, factors contributed to students' developmental unpreparedness. She speculated that teachers' lack of content knowledge and pedagogical knowledge at the early grade levels contributed to students not being ready to learn mean, median, and mode with understanding.

Further backing for warrants about students' developmental readiness came in a variety of forms. Ingrid and Joseph used future-oriented arguments to support the idea that it was not important for students to learn to use mean, median, and mode at an early age. Ingrid used the students' anticipated trajectories in school mathematics as support, suggesting that such understanding was not important for students to develop in order to succeed in algebra. Joseph felt that only some students would need to distinguish among mean, median, and mode in their future careers, so that doing so was not important for all students early on. Felicia and Heather's warrants were backed by the argument that procedural understanding should precede conceptual understanding. The two of them felt that learning how to compute the measures was possible for young students but learning them with understanding was not. Felicia said that "It is fine to have them (students) become familiar and even well versed in the methods of finding each measure," and Heather argued for building mechanical understanding first and teaching for understanding afterward. The idea of teaching mechanics in absence of conceptual understanding did not, however, go unchallenged. Joseph took issue with the idea by arguing that no lasting learning occurs when procedures are taught in isolation, and that procedural-only instruction often results in teachers offering "tricks" that students forget upon moving to another teacher's classroom.

Among those participating in the conversation, Danielle was the only one to strongly disagree with the claim that young students should not learn mean, median, and mode with understanding. She claimed that young students should learn the measures with understanding,

offering her observations of teachers' practices as evidence: "The problem is that educators have so changed the teaching sequence to reflect the [state curriculum] that we many times are holding our students back." She provided further support for her claim by stating, "I don't believe that the students can't do [it]. I am sorry, I just think they need a lot of guidance in the beginning and eventually it will be like second nature." She added that problem contexts students understand, like sports and grades, would make it possible for young students to understand the concepts of mean, median, and mode.

Elaine expressed a claim that fell between the two opposing views of not teaching for understanding and teaching all students for understanding. She felt that young students may be able to learn with understanding in some cases. She interpreted the data from the article and her own personal experience teaching students to suggest that students as young as sixth grade were capable of learning with understanding, but that some students may not be developmentally ready to do so at that age. She questioned some students' ability to learn with understanding by stating,

> Sixth grade students do have a problem determining which measure of central tendency is most appropriate for a given set of data… Is it too much for us to assume that an eleven year old will be able to grasp such a concept?

Elaine also held the theory that procedural knowledge could be developed before conceptual knowledge. She spoke of "exposing" students to the procedures for determining mean median and mode as a foundation to future learning of the meanings of the concepts, believing that "with time, conceptualization will come."

4.2 *The Role and Influence of the State Curriculum*

All of the teachers involved in the conversation had the responsibility of helping students attain the goals of the state curriculum. Danielle, Betty, Heather, Joseph, Elaine, and Felicia all claimed that changes were needed to the curriculum if students were to learn with deeper understanding. Danielle and Elaine cited their failure in helping students understand which measure of center is best for different situations as evidence the curriculum needed to be changed. Danielle, for instance, offered the following observation:

> I think back to my past year seventh graders and it was very difficult to have a discussion in class of why - if they chose mean, why; median, why; mode; why. Instead of really thinking about it I found that the students were quick to change their answer if I asked them to defend.

Heather and Joseph offered their personal frustrations with the curriculum as evidence that it should be changed. They were particularly frustrated that the curriculum dictated teaching algebra and data analysis during the same year. Heather stated, "Why on earth are these TWO separate core learning goals…and yet we combine them into one course typically taught in the first year of high school or even the year before?" Joseph affirmed the statement by citing his disagreement with the combination of the subjects into one course as well.

Several different justifications were offered for interpreting student failure to learn and teacher frustration as data to suggest the state curriculum needed to be changed. Danielle, Betty, Joseph, Elaine, and Heather forwarded the assertion that the curriculum encouraged superficial teaching and learning. All of these teachers felt that the curriculum had too many objectives for any given year. Elaine summed up this position in stating, "With such strict timetables to adhere

to, I think many topics are taught without really delving into the concept." The other teachers who felt that curriculum had too many topics in a given year also said that a reduced number of objectives would allow teachers to focus on teaching for understanding. Heather and Joseph discussed how making a separate course for data analysis would allow for focus on understanding. Heather thought that such a course would allow teachers to delve into "the whys and whens…while extending these onto broader applications outside of traditional academics." Joseph thought the algebra courses would also be made stronger in the process because teachers would have more time to "focus on the foundational aspects of the mathematics." The prevalent theme running through all of the recommendations for changes to the state curriculum was a belief in the need for more time to teach for understanding.

Some teachers made claims contradicting the idea that the state curriculum was in need of improvement. Adam, Candace, Ingrid, and Joseph (somewhat contradicting his own earlier claims) conjectured that the existing curriculum was not in need of change, but rather that its implementation needed improvement. Adam supported the idea of teaching grade level curricula four months earlier than prescribed. He cited his own experiences doing so with students, noting that the approach helped provide time for more in-depth teaching and time to teach all of the topics included on the state test. The other teachers claiming that the curriculum was not in need of change believed that more conversations among teachers of various grade levels would help improve continuity from one grade level to the next. Candace cited her experiences providing professional development in her district to support this idea, Ingrid cited experiences working with teams of teachers across grade levels, and Joseph cited his desire for more cross grade level collaboration.

Support for interpreting the evidence to indicate that the state curriculum was not in need of change came in a variety of forms. Candace felt the blame for students' failure to learn the curriculum fell to teachers, stating, "I think that if each of us took serious our responsibility at each level and covered in depth what is expected, then the people at the end of the line won't be overwhelmed." She gave further backing to this assertion by characterizing the curriculum as "hands on," "visual," and "spiraling." Ingrid characterized the curriculum as having reasonable expectations that were possible to implement in classroom settings. Candace and Joseph spoke of the curriculum as a means for mapping out instruction across grade levels, hence providing a tool for conversations among teachers of different grade levels.

4.3 *Claims Supporting Content-Specific Teaching Strategies*

Some of the claims about ways to improve the teaching of mean, median, and mode focused on content-specific teaching strategies to be used. Adam and Felicia forwarded the idea that box plots should be used to help students learn how to choose an appropriate measure of center for a given situation. Adam cited experiences using box plots with his own students to help them see where mean, median, and mode fall in the distributions of the data sets that produce them, and Felicia mentioned having her own students construct human box plots in the school hallway. Felicia characterized the use of box plots as being effective because, "The link can then be made to show how the mean is effected by an outlier and therefore can be misleading in its message." She also felt that a box plot "shows the meaning of distribution and the story that data tells." Adam characterized box plots as "physical representations," and felt that explained why they were effective for teaching how to choose among mean, median, and mode. He stated, "They seemed to make the connection better with the box and whisker plot to determine what measure of central tendency to use because it was a physical representation for them."

Ingrid, Betty, and Heather claimed that measures of center should be taught by using data sets from contexts relevant to students. Ingrid shared her experiences using grades and salaries to help students become sensitive to outliers. She felt that this provided a starting point for "interesting class discussion," because it helped explain why one would need to know the concepts. Betty discussed SAT scores as a context for data analysis. She did not refer to directly to teaching mean, median, and mode, but rather to box plots, stating,

> As far as box and whisker plots and an application of their use, it dawned on me as my son was applying for college that the SAT scores given as being typical of the students at a particular college, was the box of the box and whisker [*sic*].

Betty felt that setting data in realistic contexts like the SAT would enhance students' learning by sparking their interest. Heather proposed using claims from advertising to help students develop statistical understanding. In making this proposal, she cited her own failures to help students learn by using business contexts they did not understand. Heather justified advertising as an effective context to use for teaching measures of center because, "statistics can be 'used correctly' (in advertising) and yet be very misleading."

Perhaps the simplest content-specific teaching strategy participants claimed needed emphasis was that content must be accurately represented for students. Betty noted that this was not always done by stating,

> I remember when I was first teaching and my daughter was in the 3rd grade…and her teachers had no idea what a stem-and-leaf plot was. As I was a new teacher who had just found out myself, I too had to pass along what I had just learned. Ingrid and everyone else? Are your students coming to you think(ing) range is a measure of central tendency rather than

variability? Are there text books that say this or are the concepts taught at the same time and students confuse them? This happens way too often.

Betty's post received responses from Candace and Adam. Candace agreed with Betty's observation that teachers confuse the concepts of central tendency and variation, stating, "The concepts are taught at the same time. The distinction is not made for the students. Some teachers think that range is a measure of central tendency themselves." All three participants agreed that this teaching strategy, though perhaps the simplest to state, would be the hardest to attain, particularly in elementary schools that expected teachers to be experts in various different content areas.

4.4 *Claims Supporting the use of Content-Independent Teaching Strategies*

A prevalent type of content-independent claim in the discussion among teachers was that vocabulary instruction needed to be improved to enhance students' learning. Heather and Betty offered their own experiences as classroom teachers as data to support such claims. Each of them suggested that the practice of helping students break words down into smaller chunks helped students understand the meanings and origins of words. Heather also emphasized the strategy of helping students understand words by examining the contexts in which they occur, stating,

> Measures of central tendency aren't just simple arithmetic calculations. This is true for ALL of mathematics beyond "simple," concrete arithmetic. Algebra and Data Analysis is a high order thinking of mathematics language, and each term must be considered in context, not just in isolation.

Betty recommended a practice she had seen another teacher use, that of having students keep a vocabulary section in a mathematics notebook.

One warrant offered for interpreting language development strategies as necessary for teaching data analysis was that they help build on students' previous knowledge. In discussing the rationale for breaking words into smaller chunks, Heather stated that such activities were "critical to helping students remember and build on previous knowledge." By putting the vocabulary in terms of words they already understood, students could use pre-existing knowledge to make sense of the terms. Betty agreed, adding further backing to the strategy by noting that students can be intimidated by vocabulary and that they need to "appreciate the fact that 'big words' often mean simple concepts." Ingrid noted that explicit strategies for vocabulary instruction did not receive enough attention in her classroom, adding more justification for using such strategies more often.

Technology was spoken of as a strategy to enhance students' learning by one teacher, Danielle. In responding to frustration expressed by others about teaching measures of center, she suggested that for some struggling students she "would probably try to bring in some level of technology." She supported this claim by with the observation, "We are working with students that many times are more technologically savvy than we are." Danielle said that teachers needed to find ways to use students' technological abilities to their advantage. Danielle did not mention any particular pieces of technology to be used, and none of the other participants in the conversation inquired about the issue. Hence, unlike the other specific teaching strategies discussed, the idea of using technology was not proposed and examined from multiple viewpoints.

Another content-independent teaching strategy recommended by just one participant was the increased use of individual remediation. Joseph stated,

One of my own personal goals for the classroom this year is to focus more on

remediation, specifically with regard to reviewing my tests. The whole push to get

through the curriculum often has me stressing and focusing on a standard and then

moving on to the next once the test is completed. I have, on occasion, done major re-

examination of my tests, but I see a much greater need to use this time more effectively.

I usually discuss the answers and concepts of the test, but I need to do more

individualized work with students to provide what they really need.

The data supporting his argument was his personal observation that he had not spent enough time

working with students individually. Support for viewing this as evidence for more remediation

was based on the assumption that he moved through the curriculum too quickly, which made it

difficult for students to correct mistakes they made on their examinations.

5. DISCUSSION

Teachers' discourse can reveal factors beyond the content knowledge of teachers and

students that make the mean, median, and mode problematic curricular concepts. This section

will focus upon hypothesizing how, in this case, content knowledge worked in concert with

knowledge of students, knowledge of pedagogy, and beliefs (Fennema & Franke, 1992) to

influence the types of claims that teachers made. Hypothetical webs of influence with teachers'

knowledge and beliefs as nodes will be shown in a causal network (Miles & Huberman, 1994) to

help explain why each pervasive claim entered the discourse. A description of each causal

network will then be given in order to identify factors pertinent to the persistent difficulties in

teaching and learning mean, median, and mode.

5.1 Pervasive Claim 1: Meanings for Measures of Center should not be taught to Young Students

Figure 2 depicts the relationships between participants' knowledge and beliefs and the claim that young students should not study the meanings for mean, median, and mode. In Figure 1, as well as in the rest of the causal networks to be considered in this report, arrows indicate influence of a type of knowledge or belief. Heavy arrows represent influences that were readily apparent in multiple participants' contributions to the conversation.

<INSERT FIGURE 1 HERE>

Teachers frequently mentioned their knowledge of students as part of the argument that young students should not study meanings for mean, median, and mode. Observations of students' difficulties learning the concepts, both in the participants' classrooms and in the article used to catalyze conversation, appeared to fuel the belief that young students were not developmentally ready to study the concepts. Although the validity of this belief was questioned during the conversation, it nonetheless heavily contributed to the pervasive claim that young students should not be asked to learn the concepts with understanding. Hence, a striking aspect of the conversation is that many teachers appeared to voice and agree upon a quasi-Piagetian justification for holding off on teaching mean, median, and mode for understanding.

Although further empirical research may be necessary to confirm or refute the quasi-Piagetian theory negotiated by some participants, empirical evidence exists to refute another source fueling pervasive claim 1. As depicted in the lower right-hand corner of Figure 1, multiple participants held the pedagogical view that it was acceptable for young students to learn procedures for mean, median, and mode, but not to learn the conceptual basis for the procedures. Several existing empirical studies illustrate that learning procedures without understanding

initially makes it more difficult to learn with understanding later on (Mack, 1990; Wearne & Hiebert, 1988). Students who are taught initially by rote exhibit cognitive and affective resistance when asked to re-learn concepts with understanding later on (Pesek & Kirshner, 2000). In accepting the pedagogical idea that it is acceptable for procedural teaching to precede conceptual, a subset of the teachers markedly parted ways with a segment of the research community.

Another factor contributing to pervasive claim 1, the belief that not all students need to learn mean, median, and mode with understanding, also conflicts with discursive themes in the research community. Some participants de-valued learning the measures with understanding due to the perception that they would not need such knowledge in their occupations or in taking algebra later on in their school careers. In the statistics education literature, the importance of statistical literacy for all students has come to the forefront in curriculum documents (Franklin et al., 2007) and empirical research (Ben-Zvi & Garfield, 2004). As noted by some of the participants in the present study, students need to be prepared to make sense of statistical claims made in venues such as advertisements. Even if they are not required to utilize statistics directly in their future occupations or schoolwork, students will not likely escape the need to make judgments about statistical claims made by others in the course of everyday events.

A final factor, relating to content knowledge, contributing to pervasive claim 1, was that teachers' lack of content knowledge provides justification for not teaching the meanings of mean, median, and mode to young students. One could consider this somewhat of a fatalistic position, or simply consider it to be highly pragmatic. As mentioned at the outset of this paper, the difficulties that students have in learning measures of center with understanding has been well documented (Mokros & Russell, 1995; Strauss & Bichler, 1988), and teachers' content

knowledge is a likely contributing factor to the difficulties (Groth & Bergner, 2006). Given such a state of affairs, it may seem reasonable to some to abandon the enterprise of teaching for understanding altogether. However, given the growing importance of statistical literacy for all students, it would seem that abandoning the enterprise altogether is morally and socially untenable, so that efforts need to continue to be directed toward answering the question of *how* to accomplish the goal rather than speculating about whether or not the goal *should* be pursued.

5.2 Pervasive claim 2: The State Curriculum Needs Revision

The claim that the state curriculum needed to be changed drew a good amount of support from participants. Figure 2 depicts the sources of support for the claim. Teachers interpreted students' struggles to learn mean, median, and mode with understanding to mean that the state curriculum was in need of revision. The teachers believed that having more instructional time available was a key to improving students' understanding. A sentiment shared among many was that more instructional time, and hence more pedagogical options, would be available if some objectives were removed from the curriculum. The focus of the teachers' curricular conversations thus largely became how the quantity of objectives influenced the quantity of time available. The assumption that more instructional time equates to enhanced student learning, which has been questioned in the empirical literature (Pesek & Kirshner, 2000), became an unquestioned working assumption for the conversation. This assumption was not questioned even by those participants who felt the state curriculum was not in need of revision.

<INSERT FIGURE 2 HERE>

One of the means proposed for reducing the number of curricular objectives was to separate objectives for data analysis from those for algebra. On the surface, such a move seems to align

with philosophical arguments in the literature that statistics should be considered a subject in its own right rather than a branch of mathematics (Cobb & Moore, 1997). Moore (1988) sparked many of the arguments for considering statistics to be a discipline in its own right, since its origins and foundational questions differ from those of mathematics. In the present study, however, the proposal to distinguish objectives for mathematics from those for statistics was justified solely in terms of paring down the number of curricular objectives teachers were responsible for teaching. Although the participants' claim that a distinction should be made between statistics and mathematics resembled claims made in the literature, the justification differed sharply.

5.3 Pervasive claim 3: Content-Specific Teaching Strategies Need Attention

Utilization of box plots, real-world contexts, and accurate representation of content for students were proposed as content-specific pedagogical strategies in need of attention. Figure 3 indicates that teachers drew upon their content knowledge to recognize that box plots reveal characteristics of distributions pertinent to choosing among mean, median, and mode as measures of center. They also drew upon content knowledge to recognize that the concepts are not always represented accurately, pointing out that teachers mistake measures of center for measures of spread. Participants' pedagogical knowledge was reflected in the ideas that box plots could be represented physically by students and that interesting contexts could help catalyze class discussions. Participants' observations of students engaging in the study of box plots and real world contexts for data analysis contributed to the belief that the two strategies helped motivate students. This belief, in turn, provided further justification for the claim that content-specific teaching strategies needed attention.

<INSERT FIGURE 3 HERE>

The teachers' claims about content-specific pedagogy at times mirrored ideas in current research literature, and at other times did not. Although it is true that box plots display the characteristics of distributions, they can also be difficult for students to interpret because they hide individual data values (Biehler, 1997). Bakker, Biehler, & Konold (2005) recommended displaying dot plots for distributions alongside box plots in order to minimize students' difficulties interpreting the displays. Hence, the claim that box plots (without accompanying dot plots) should be used as a way to help students decide among mean, median, and mode departed a bit from current recommendations based on empirical research. The claim that teachers often lacked the content knowledge necessary for teaching statistics, however, aligned with empirical research (Groth & Bergner, 2006). The claim that realistic contexts familiar to students should be used for data analysis activities also closely resonated with themes in current literature (Groth, 2006b), although difficulties students may encounter in interpreting data in context were not dealt with in detail by the participants.

5.4 Pervasive Claim 4: Content-Independent Teaching Strategies Need Attention

Whereas one set of claims focused upon content-specific pedagogy, another set of claims focused on the idea that content-independent teaching strategies needed attention. As shown in Figure 4, this type of claim was characterized by its detachment from justifications that drew upon participants' knowledge of content. Instead, participants drew upon their knowledge of students' characteristics to justify their recommendations. The observation that students have extensive experiences using technology outside of school and that they have different individual needs were used to justify the content-independent strategies of technology usage and

remediation, respectively. Some participants' beliefs that students are intimidated by vocabulary led them to draw upon pedagogical knowledge of the teaching strategies of breaking words into smaller chunks and having students keep vocabulary notebooks. The availability of these pedagogical tools, in turn, helped justify more attention to implementing content-independent teaching strategies.

<center><INSERT FIGURE 4 HERE></center>

Although content-independent teaching strategies can be powerful because of their abilities to cut across content areas, the content-independent claims made by participants could generally have been stronger and more pertinent to teaching mean, median, and mode if contextualized in terms of the content area of statistics. The generic claim that students' familiarity with technology could be drawn upon to enhance their learning could have been strengthened with reference to content-specific technology, such as applets that allow one to observe how adding or removing values to a data set influences its accompanying measures of center (Velleman, 2003). The idea that students' individual needs should be addressed could have been strengthened with discussion of the specific levels of thinking individual students have exhibited about averages in empirical research (Watson & Moritz, 2000). The discussion of vocabulary strategies could have been enhanced with discussion of the meanings students attribute to the word "average" (Watson & Moritz, 2000), as well as how the terms mean, median, and mode are sometimes strategically referred to as "averages" to help individuals bolster arguments (Huff, 1954).

6. CONCLUSION

The case examined in the present study illustrated that discourse among teachers can have instances of divergence as well as convergence with themes present in the discourse among those

engaged in formal empirical research in statistics education. Themes present in both discourses included the need to distinguish between mathematics and statistics, the need for enhanced teacher content knowledge, and the need for students to reason about contextualized data. Divergence between the discourses, however, was apparent when teachers began to discuss their justifications for claims about improving teaching. Some of the teachers' justifications were not as unproblematic as portrayed in the conversation, such as the assumption that more time on task equates to more student learning, and also the portrayal of contextualized data as somewhat of a panacea for statistical learning. Increased overlap between the discourse of formal empirical research and the teachers' more informal conversation could have been beneficial in such cases, since existing empirical research has the potential to raise doubts about such frequently held "common sense" assumptions about teaching and learning.

Increased overlap between teachers' informal discourse and the discourse of empirical research can also be beneficial for researchers in statistics education. Knowing specific concerns that arise in the discourse among teachers can help inform the substance of research questions. Discourse among teachers in the present study, for instance, suggested that research examining specific connections between Piaget's developmental theory and students' learning of mean, median, and mode has the potential to answer practice-based questions about students' readiness to learn the concepts with understanding. Knowing of specific concerns that arise in the discourse among teachers can also help inform the substance of curricular documents. Curricular standards written specifically to address statistics (Franklin et al., 2007) are a relatively new phenomenon, so attending to teachers' discourse is particularly vital to anticipating conversational themes curricular recommendations may spark in teachers' conversations. As

curricular recommendations are shaped by empirical research, points of emphasis about the research to make for teacher audiences can be honed by closely attending to their conversations.

In conclusion, the present study can be viewed as an example of the result of paying close attention to teachers' conversations about teaching a specific statistical concept. As researchers continue to attend to such conversations, valuable direction for forming research questions and writing curricular recommendations can be gained. The rapidly-growing discourse community of statistics education researchers can become more conversant with teachers' concerns about teaching statistics in the process. The present study contributes to the process of making the statistics education research community more conversant with teachers by providing a working framework for identifying and analyzing the themes that may occur in teachers' conversations about teaching mean, median, and mode, along with a general strategy for catalyzing such conversations among teachers.

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    knowledge. *Mathematics Teacher Education and Development*, *6*, 3-18.

Table 1

Teachers Participating in the Conversation

Participant pseudonym	Grade levels taught	School Affiliation	Number of posts made to discussion
Adam	7	District A	4
Betty	8	District B	6
Candace	4-6	District B	4
Danielle	7	District C	4
Elaine	6	District D	3
Felicia	4-8	District B	4
Heather	7-12	District E	10
Ingrid	8	District B	4
Joseph	8	District B	5

Table 2

Types of Claims Made by Participants

Type of claim	Number of teachers making the claim
Meanings for measures of center should not be taught to young students	4
Meanings for measures of center should be taught to young students	1
Meanings for measures of center might be taught to young students	1
The state curriculum should be changed to facilitate learning	6
The state curriculum should be better implemented to facilitate learning	4
Meanings for measures of center should be taught using box plots	2
Meanings for measures of center should be taught using meaningful contexts	3
Meanings of measures of center should be accurately represented by teachers	3
Vocabulary development is a critical issue	4
More use of technology is necessary	1
More remediation is needed for students	1

Figure Captions

Figure 1. Causal network for pervasive claim 1

Figure 2. Causal network for pervasive claim 2

Figure 3. Causal network for pervasive claim 3

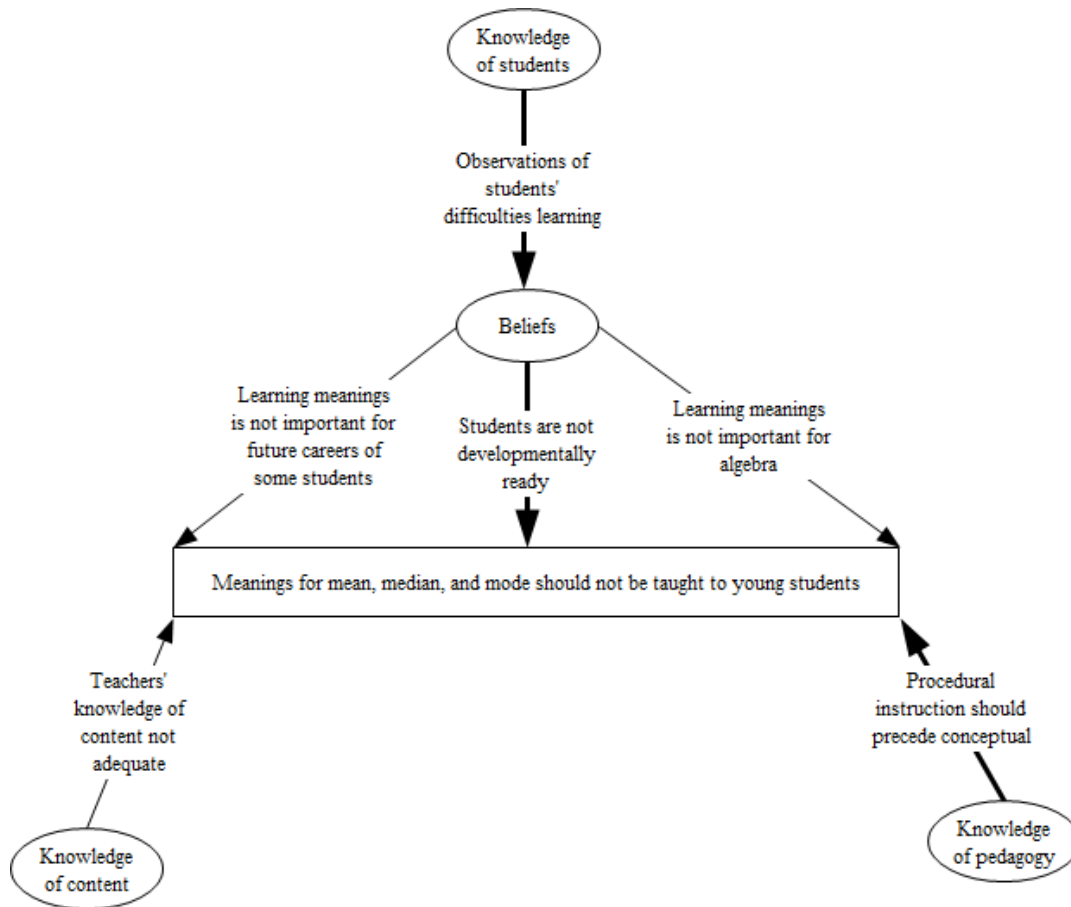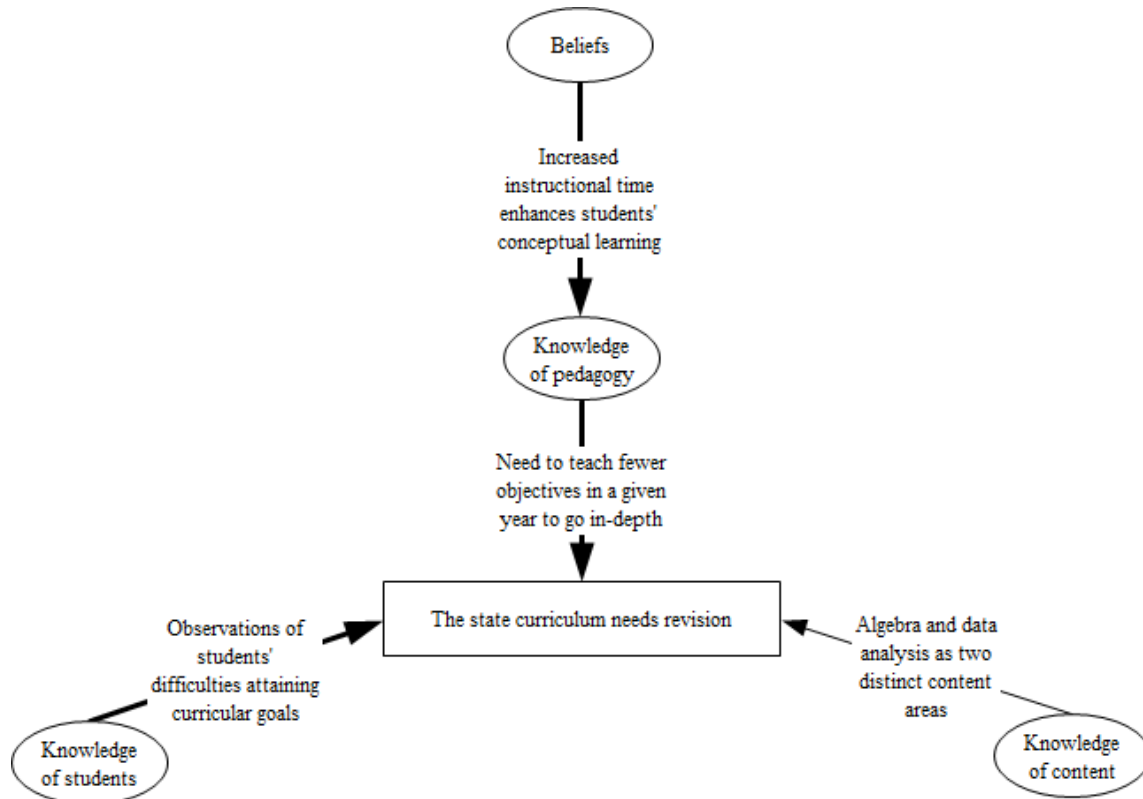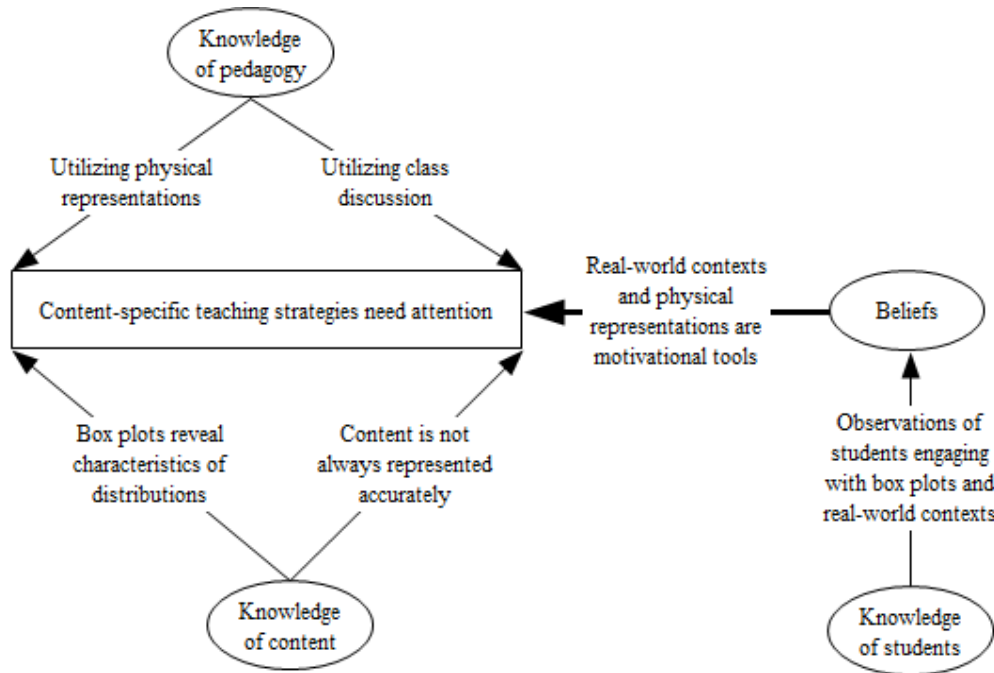*Figure 4*. Causal network for pervasive claim 4

Figure 1

Figure 2

```
                        ┌──────────┐
                        │  Beliefs │
                        └──────────┘
                             │
                        Increased
                        instructional time
                        enhances students'
                        conceptual learning
                             │
                             ▼
                        ┌──────────┐
                        │ Knowledge│
                        │of pedagogy│
                        └──────────┘
                             │
                        Need to teach fewer
                        objectives in a given
                        year to go in-depth
                             │
                             ▼
         ┌─────────────────────────────────────────┐
         │  The state curriculum needs revision     │
         └─────────────────────────────────────────┘
   Observations of                      Algebra and data
   students'                            analysis as two
   difficulties attaining               distinct content
   curricular goals                     areas

   ┌──────────┐                         ┌──────────┐
   │ Knowledge│                         │ Knowledge│
   │of students│                        │of content │
   └──────────┘                         └──────────┘
```

Figure 3

Figure 4