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The contribution of a large baseline intersatellite link to relativistic metrology

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Abstract— The precise estimation of helio- and fundamental physics parameters is essential for an accurate modeling of the solar system. The enhancement of radio tracking performances will enable unprecedented measurements of celestial mechanics, approaching the ultimate limits attainable with a single planetary mission. We report in this study the results of our numerical simulations with a novel interplanetary mission configuration composed of two probes at different planets able to establish, apart from a classical ranging link with the Earth, an inter-satellite ranging link. It is shown how this configuration allows constraining planetary positions much more tightly compared with the case of Earth-based measurements only, leading to a very precise retrieval of fundamental and heliophysics parameters. Through the analysis of an extensive set of simulations we investigate the applicability and the beneficial effects that a large baseline intersatellite link introduces in the estimation process and we show that such a mission concept performs better than any other single-probe planetary mission with the same measurement accuracy. In particular we demonstrate that over short timescales (<2 years) the gain in terms accuracy in the retrieval of the parameters of interest is considerable and might lead to unprecedented improvements in terms of fundamental physics and solar system modeling.

Keywords—*Test of General Relativity, Solar System Expansion, Radio tracking systems*

I. INTRODUCTION

The precise estimation of helio- and fundamental physics parameters that include the Nordtvedt parameter (η), the temporal variation of the solar gravitational constant ($\dot{\zeta} = \frac{\dot{\mu}_\odot}{\mu_\odot}$), the Compton wavelength for the graviton (λ_G), and coefficients (e.g., β and γ) of the Parameterized Post-Newtonian (PPN) formalism for Einstein's theory of General Relativity represents an essential piece of information to model accurately the solar system evolution. A precise estimation of these parameters has become possible recently thanks to the great enhancement of spacecraft radio tracking performances. Important results have been achieved with past

planetary missions, as reported in Table 1, and further developments are expected in the near future from the MORE experiment of the BepiColombo mission that will yield the ultimate limits of radio tracking systems [4]. Further improvements in the estimation of these parameters through a planetary mission is limited by two main reasons: (1) the correlation between the adjusted parameters due to the orbital geometry, (2) the requirement to perform radio tracking measurements over long time scales ($>3-4$ years at least).

To overcome these limitations Smith et al. [5,6] have proposed a novel mission concept, Trilogy, whose 3-planets ranging configuration offers the possibility to conduct, for the first time, non-Earth-centered metric measurements in a varying but closed triangle configuration. The scope of this work is to analyze and simulate the Trilogy mission scenario to assess the accuracies of helio- and fundamental physics parameters estimation.

TABLE I.

Parameter	Value	Mission	Source
η	$(-6.6 \pm 7.2) \times 10^{-5}$	MESSENGER	[1]
$\beta - 1$	$(-1.6 \pm 1.8) \times 10^{-5}$		
ζ	$(-6.13 \pm 1.47) \times 10^{-14}$		
$J_{2\odot}$	$(2.246 \pm 0.022) \times 10^{-7}$	Cassini	[2]
$\gamma - 1$	$(2.1 \pm 2.3) \times 10^{-5}$		
λ_G	$> 2.7 \times 10^{14} \text{ km}$	MRO ¹	[3]

II. CONTRIBUTION

A. Orbit Determination

The estimation of physical parameter by using spacecraft radio tracking is done through orbit determination (OD) processing. The OD process relies on the comparison between actual radiometric measurements (*observed* observables) and

¹ λ_G is a theoretical estimate, as pointed out in [3], based on the estimated values of γ and β . All the other value shown in the table were obtained from reduction of radio tracking data.

the prediction of these measurements based on a dynamical model that describes the motion of the spacecraft (*computed* observables). If all the forces and the physical phenomena that drive the solar system motion were perfectly known the only difference between the *observed* observables and the *computed* observables would consist in the measurement noise due to the electronics. When some effects are not perfectly modeled remaining uncompensated errors stand out. The scope of the OD process is the minimization of these residual discrepancies through the fitting of the dynamical model parameters to the observations. This tool is the key to retrieve precise estimates of all the poorly known physical parameters that have an influence on the motion of the space probe. The solution of the OD problem can be obtained using the well-known linearized minimum variance least-squares filter with *a priori* information [7], whose solution is:

$$\hat{\mathbf{x}} = (H^T R^{-1} H + \bar{P}^{-1})^{-1} (H^T R^{-1} \mathbf{y} + \bar{P}^{-1} \bar{\mathbf{x}}) \quad (1)$$

where $\hat{\mathbf{x}}$ is the vector of differential corrections to the model parameters (*state*), H is the *design matrix* that contains the partial derivatives of the observables with respect to the state, R is the covariance matrix of the measurement error, \mathbf{y} are the observations, \bar{P} and $\bar{\mathbf{x}}$ are the *a priori* covariance and estimate of the state, respectively.

B. Trilogy Concept

The effects of parameters such as η or ζ on the motion of the solar system planets are detectable with the latest radio tracking systems and techniques and many advances and important discoveries have been done in the past years. However, a better knowledge of these forces may require measurements that are not only Earth-centered. A single ranging measurement locates the probe on a sphere centered in the observer with a radius corresponding to the range measurement. This ambiguity is typical of this kind of measurement and can be reduced by combining many measurements. However, a ranging system provides much more precise positioning in the observer-probe line-of-sight (LOS) than other directions. Moreover, if all the measurements are Earth-based the estimates of the parameters would be strongly correlated to Earth position possibly leading to degeneracies in the dynamical model. The Trilogy mission concept is based on the introduction of measurements that are not Earth-centered to break or, at least, to reduce those correlations. Trilogy consists in a closed triangle radio tracking configuration whose vertices are two probes, orbiters or landers, and an Earth Deep Space Network (DSN) station. This configuration includes radiometric ranging measurements between the three planets involved allowing the precise retrieval of their motion. Three different configurations have been simulated:

- Mercury-Earth-Mars
- Mercury-Earth-Jupiter

- Venus-Earth-Mars

The analysis of a large baseline inter-satellite link together with Earth-based ranging measurements empowers strongly the observation geometry leading to a much more precise determination of planetary ephemerides in the directions orthogonal to the Earth-planet LOS. Fig. 1 shows the Mercury position uncertainty ellipses in the LOS frame² obtained in a 6-months Mercury-Earth-Jupiter configuration. We can observe the great benefits of the employment of the Mercury-Jupiter link, which contributes mostly to reduce the uncertainty in the position of Mercury in the normal direction. Clearly, a better constraint on the position of the planets involved allows a better estimation of the dynamical parameters that drive their motion. A secondary, but not negligible, advantage of such a measurement setup is the reduction of the mission duration (compared to a single-probe mission) necessary to obtain a certain level of precision on the estimation of the planets' state.

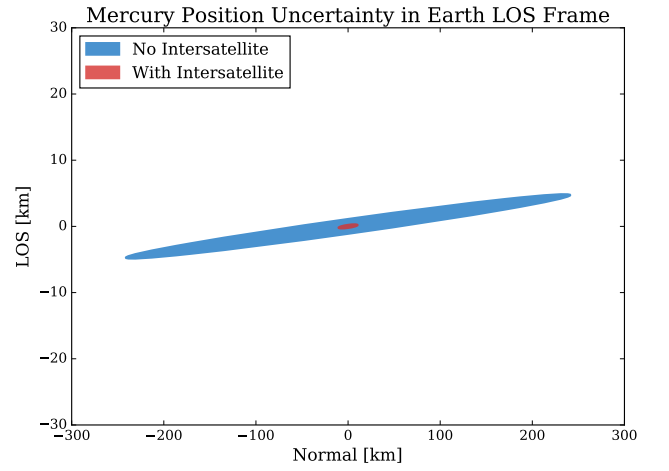


Figure 1 - Mercury-Earth-Jupiter configuration: Mercury position uncertainty ellipses ($1-\sigma$) obtained in the presence (red ellipse) and in absence (blue ellipse) of the Mercury-Jupiter intersatellite link. The ellipses are expressed in the line-of-sight (LOS) frame.

C. Dynamical Model

The dynamical model adopted in this work includes the mutual gravitational interaction between all main solar system bodies accounting for General Relativity (GR) effects, the oblateness of the Sun, the Earth and Jupiter. We estimated the state (position and velocity) of the probes, the ephemerides and gravitational constants of the Earth and the two other planets involved in the measurements, the gravitational constant of the main solar system bodies, and the following parameters:

- The Eddington parameters γ and β . These parameters are unity in GR. The parameter γ measures the space-time curvature while β is related to the non-linearity in the superposition law for gravity [8]. In this work the preferred frame

² For this kind of analysis the LOS frame is more suitable than the inertial one. This coordinate frame is a rotating coordinate frame defined by a body in motion relative to a center body. The position direction of the body relative to the center is used for the X-axis. The Z-Axis is defined as

the angular momentum vector of the body relative to the center and the Y-axis (also referred to as *Normal*) closes the right-handed orthonormal triad.

parameters α_i have been considered to have a null value.

- The Nordtvedt parameter η , describing the violation of the strong equivalence principle, i.e. the difference between the inertial and gravitational mass. The relation between the two kind of masses can be expressed as:

$$\mu^I = (1 - \eta\Omega)\mu^G$$

where μ^I and μ^G are the inertial and gravitational masses (times G) respectively and Ω is the gravitational self-energy [9]. The Nordtvedt equation has been included as a constraint in the estimation process. The Nordtvedt equation constrains the PPN parameters to be related as:

$$\eta = 4\beta - \gamma - 3$$

- The linear time variation of the Sun's gravitational constant ζ .
- The Compton wavelength for the gravity field λ_G associated to the Yukawa-like gravitational potential [3]. If the gravitational field is massive the gravitational potential has the form:

$$U = \sum_i \frac{GM_i}{r_i} \exp\left(-\frac{r_i}{\lambda_G}\right)$$

The partial derivatives (needed for the design matrix) of the ranging measurement with respect to all these parameters have been computed and implemented in the numerical integration of the equation of motion. As an example, Figure 2 shows the partial derivatives of the range with respect to ζ in the Mercury-Earth-Mars configuration.

The computation of the partial derivatives apart from being required for the OD solution enables the quantification of the amount of information brought by a certain link. One can define the power of the partial derivative signal as:

$$Q_p = \sum_k \left(\frac{\partial \rho(t_k)}{\partial p} \right)^2 \quad (2)$$

Where $\partial \rho(t_k)/\partial p$ is the partial derivative of the ranging measurement with respect to the p -th component of the state at time t_k and the summation is intended over the entire time span of the mission $k = 1, \dots, N$, with N the number of measurements. Being Q_p one of the diagonal elements of the Fisher information matrix, this quantity corresponds to the amount of information on the parameter p contained in the ρ measurement. The power of the signal can be used to perform a first analysis of the impact of a specific geometry on the estimation of the parameters of interest.

D. Numerical Simulations

The simulation of an OD experiment requires to simulate the *observed* observables. Several strategies are possible depending on the scope of the simulation. The simulations presented in this work are intended for a formal uncertainty and covariance analysis thus the *observed* observables are derived from the same dynamical model employed for the *computed* observables. The difference between the two sets of observations consists in the different implementation of the measurement noise. Our numerical setup simulates a realistic scenario that is useful to evaluate the estimation performances on the *state* parameters.

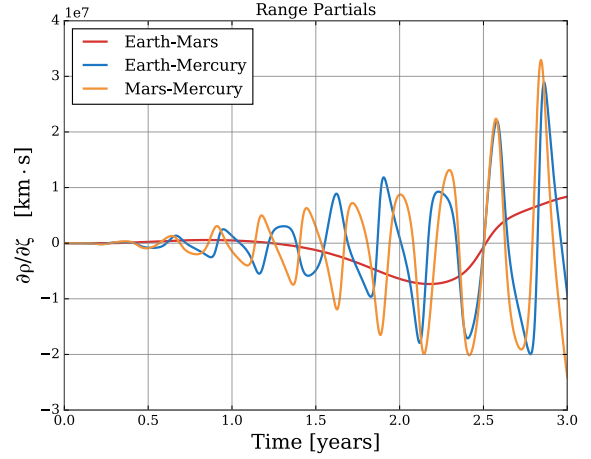


Figure 2 - Range partial derivatives with respect to ζ obtained in a 3-year Mercury-Earth-Mars configuration.

The simulations we are presenting are based on the fundamental assumption that the orbits of the probes about the relative central planets have been resolved, employing the doppler data, thus allowing the analysis of the sole ranging data. This is a realistic assumption since, as exposed in the original mission concept [5], the Trilogy network would be suited not only for fundamental physics studies but also for “local science” investigations (e.g. gravity field and rotational state of the hosting planet) that require a precise determination of the orbit of the probe. This results in a normal point reduction (for the definition of normal points see [11]).

We simulated the three setups presented in section II.B. It is assumed that the measurements are collected every 10 hours and that the motion of the planetary probes is measured with an accuracy of 20 cm (derived from the single-shot accuracy of the Advanced Ranging Instrument (ARI) of BepiColombo [10]) and that that an inter-satellite ranging system with a comparable accuracy is available. Note that this is a worst-case assumption since in a real mission scenario much better performances (corresponding to a noise reduction up to an order of magnitude) can be obtained integrating over 10 hours ARI measurements. A white Gaussian noise with a standard deviation corresponding to the assumed accuracy has been superimposed on the raw measurements.

The measurement generation accounts for occultation events of all the solar system bodies and all the data too heavily affected by solar plasma noise (i.e. in proximity of superior solar conjunctions when the radio photons pass near the Sun with $b < 7R_\odot$ where b is the impact parameter of the radio link and R_\odot is the Sun radius) were discarded. The DSN Earth station chosen is the Goldstone Deep Space Station (DSS) 25 for which an elevation threshold of 15° has been set. The *a priori* values on planetary positions and gravitational constants considered in the OD filter are derived from the latest solutions reported in the literature, no *a priori* values have been assumed for the parameters listed in Table 1. This allows an unconstrained analysis of the estimation performances on the parameters more strictly tied to the fundamental physics interpretation in order to avoid any kind of influence of possible and unknown systematic errors in previous estimates.

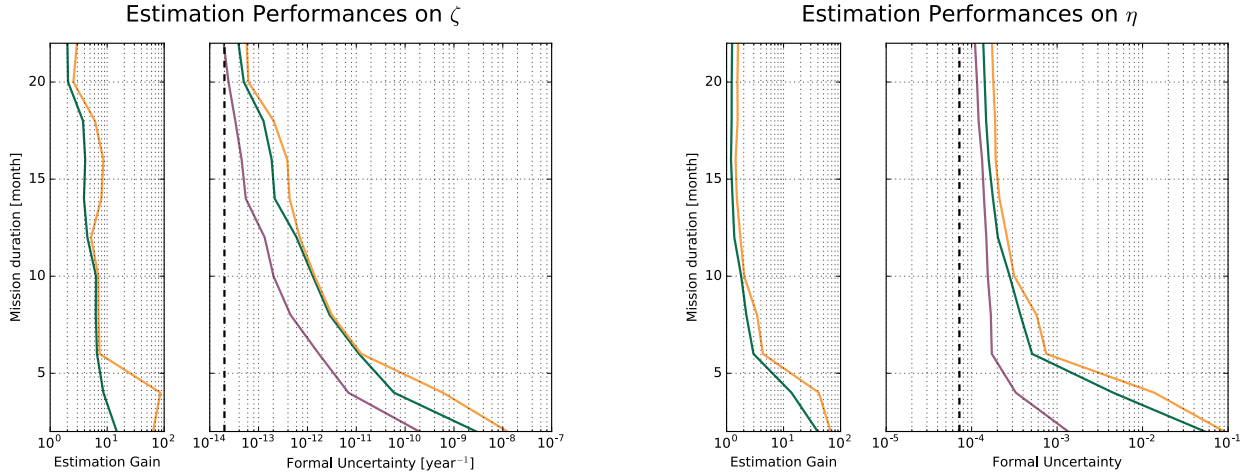


Figure 3 – Estimation gain and formal uncertainty for ζ (left figure) and η (right figure). In the right panel of each figure we represent the formal uncertainty attainable in the Mercury-Earth-Mars configuration as a function of time. The purple line corresponds to the Trilogy configuration, the green line to the Trilogy with no intersatellite link configuration and the yellow line to a single Mercury orbiter. The black vertical dashed line represents the current uncertainty limit, as reported in Table I. The left panel of each figure presents the same results in terms of estimation gain of Trilogy with respect to a single Mercury orbiter (yellow line) and to the no-intersatellite link configuration (green line).

III. RESULTS

We can define the *estimation gain* as the ratio between formal uncertainties obtained in different mission configurations. The analysis of the estimation gain variation with respect to the mission duration gives an insight on the applicability of the Trilogy measurement concept and on the contribution of the large baseline inter-satellite link on the global solution. Figures 3 shows the estimation results obtained on η and ζ in the Mercury-Earth-Mars configuration, but analogous results can be obtained on all the other parameters and configurations. These results were recovered by means of an OD solution with no *a priori* information to enable an unconstrained analysis. The figure shows that the inter-satellite link provides a significant contribution to the OD solution for short-time missions demonstrating the beneficial effects of this additional link on constraining planetary positions. For long-time missions the estimation gain tends to an asymptotic value of $\sim\sqrt{N}$ with $N = 3$ for the estimation gain associated to a single planetary orbiter (yellow line) and $N = 3/2$ for the one associated with a Trilogy configuration without intersatellite link (green line). These asymptotic values correspond to the mere contribution given by the increase of the number of measurements. Our results suggest that Trilogy can provide substantial improvements in the determination of planetary orbital dynamics in the short time scale leading to estimates that are far more accurate than any other single-probe or two-probe mission capable of the same measurement performances. For comparison, to obtain the same results of a 5-months Trilogy mission on η we would need at least 10 months of “traditional” missions and conversely a 5-months Trilogy mission would lead to a formal uncertainty on η an order of magnitude lower than a single Mercury orbiter. As stated before the same considerations are valid for all the other considered parameters. However as reported clearly in Figure 3 a short

mission duration might not be sufficient, with the measurement quality we assumed, to attain an unprecedented accuracy level on the parameters we are interested in³. The analysis of the estimation quality as a function of the mission duration and the previous considerations about the asymptotical decrease of the estimation gain suggest that a further enhancement of these results has to be obtained through a better measurement system rather than with a longer mission, being the formal uncertainty related linearly with the measurement noise. It is important to underline that we assumed a conservative level of noise (20 cm @ 10h), that is at least by a factor of 10 higher than the expected noise for BepiColombo (as explained in section II.D). It is then interesting to report on simulations performed assuming a more realistic level of noise. Table 3 reports the results obtained with a 2 cm noise level, which represents our expectation for BepiColombo measurements integrated over 8 h and is compatible with a lower bound for future laser ranging data, we have assumed a 1.5-year mission duration. We can see that in this case a 1.5-year Trilogy mission might lead to an unprecedented level of accuracy on helio- and fundamental physics parameters. In Figure 4 we report in a more concise way the results expressed in Table 3 as the ratio between the attainable accuracy and the current uncertainty limit. Figure 4 depicts a complex scenario where the selection of the best configuration between the three we have simulated strongly depends on the objectives of the investigation.

IV. CONCLUSION

We presented our results regarding helio- and fundamental physics parameters determined with a novel mission concept (Trilogy) based on inter-satellite tracking between deep space probes. Our estimates were obtained through an extensive set of OD simulations performed over three different geometrical configurations. The analysis suggests that a mission based on

³ Current limits on the parameters relevant to this work are reported in Table I. The best current estimate for μ_{\odot} is $132712440042.2565 \pm 0.35 \text{ km}^3\text{s}^{-2}$ [1].

the Trilogy concept performs better than any other single-probe planetary mission capable of the same ranging measurement accuracy on short time scales. The estimation gain attainable with such a mission configuration reaches considerable values with mission durations < 2 years, configuring Trilogy as a valuable mission concept to attain breakthrough advances in the modelling of solar system dynamics without the need of long-duration missions. If such a measurement system is available, a 1.5-year mission with the same measurement accuracy as BepiColombo would lead to unprecedented improvements in the knowledge of fundamental physics parameters paving the way to pivotal discoveries in the field of fundamental physics and solar system modeling.

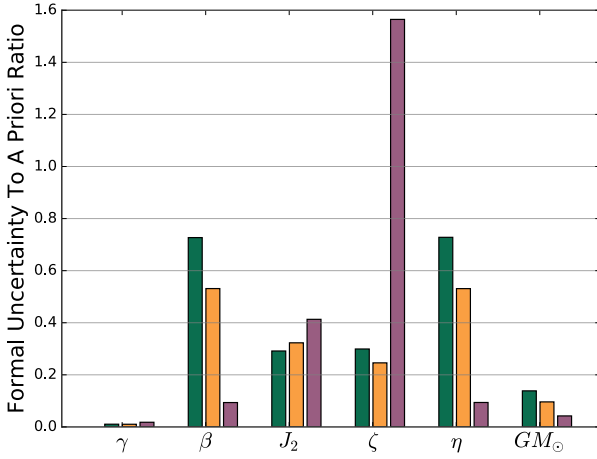


Figure 4 – Ratio between formal uncertainties and current accuracy limit in the three simulated configurations. The green bars are relative to the Mercury-Earth-Mars configuration, the yellow bar to the Mercury-Earth-Jupiter and the purple one to the Venus-Earth-Mars one.

TABLE II. SIMULATION RESULTS

Parameter	Mercury-Earth-Mars	Mercury-Earth-Jupiter	Venus-Earth-Mars
γ	2.3×10^{-7}	2.3×10^{-7}	4.1×10^{-7}
β	1.3×10^{-6}	9.5×10^{-6}	1.7×10^{-6}
$J_{2\odot}$	6.4×10^{-10}	7.4×10^{-10}	9×10^{-10}
ζ [yr^{-1}]	5.9×10^{-15}	4.9×10^{-15}	3.3×10^{-14}
η	5.2×10^{-5}	3.8×10^{-5}	6.7×10^{-6}
μ_\odot [km^3s^{-2}]	4.8×10^{-2}	3.3×10^{-2}	1.4×10^{-2}
λ_G [km]	$> 1.4 \times 10^{15}$	$> 3.8 \times 10^{15}$	$> 1.2 \times 10^{15}$

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