A PRIORI VOTING POWER AND THE U.S. ELECTORAL COLLEGE

Nicholas R. Miller
Department of Political Science
University of Maryland Baltimore County (UMBC)
Baltimore, Maryland 21250
nmiller@umbc.edu

August 2009

Abstract

This paper uses the Banzhaf power measure to calculate the *a priori* voting power of individual voters under the existing Electoral College system for electing the President of the United States, as well as under variants of this system in which electoral votes are either apportioned among the states in a different manner or cast by the states in a different manner. While the present winner-take-all manner of casting state electoral gives a substantial advantage to voters in the largest states, this advantage is diluted by the small-state advantage in apportionment. Moreover, most of the alternative Electoral College plans that have been proposed to remedy this large-state advantage give an equally substantial voting power advantage to voters in small states. Direct popular election of the President uniquely maximizes and equalizes individual voting power.

Key words: Electoral College, voting power, Banzhaf measure

Prepared as a chapter in a festschrift volume edited by Manfred Holler and Mika Widgren in honor of Professor Hannu Nurmi of the University of Turku. Earlier versions of portions of this paper were presented at Creighton University, the London School of Economics, the 2008 Annual Meeting of the Public Choice Society, and the 2008 Annual Meeting of the Institute for Operations Research and the Management Sciences. I thank Dan Felsenthal, Moshe Machover, and especially Claus Beisbart for very helpful criticisms and suggestions.

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1. *Introduction*

The President of the United States is elected, not by a direct national popular vote, but by an Electoral College system in which (in almost universal practice since the 1830s) separate state popular votes are aggregated by adding up electoral votes awarded on a winner-take-all basis to the plurality winner in each state. State electoral votes vary with population and at present range from 3 to 55. The Electoral College therefore generates the kind of weighted voting system that invites analysis using one of the several measures of *a priori* voting power. With such a measure, we can determine whether and how much the power of voters varies from state to state and how voting power would change under various alternatives to the existing Electoral College system.

With respect to the first question, directly contradictory claims are commonly expressed. Many commentators see a substantial small-state advantage in the existing system but others see a large-state advantage. Partly because the Electoral College is viewed by some as favoring small states and by others as favoring large states, it is commonly asserted that a constitutional amendment modifying or abolishing the Electoral College could never be ratified by the required 38 states. The so-called "National Popular Vote Plan" (an interstate compact among states with at least 270 electoral votes that would pledge to cast their electoral votes for the "national popular vote winner") has been proposed as a way to bypass the constitutional amendment process.

The divergent assessments of bias in the Electoral College often arise from a failure by commentators to make two related distinctions. The first is the theoretical distinction between *voting weight* and *voting power*. The second is the practical distinction between *how electoral votes are apportioned* among the states (which determines their voting weights) and *how electoral votes are cast* by states (which influences their voting power).

These distinctions were clearly recognized many years ago by Luther Martin, a Maryland delegate to the convention that drafted the U.S. Constitution in 1787. Martin delivered a report on the work of the Constitutional Convention to the Maryland State Legislature in which he made the following argument.

> [E]ven if the States who had the most inhabitants ought to have the greatest number of delegates [to the proposed House of Representatives], yet the *number of delegates* ought not to be in *exact proportion* to the *number of inhabitants* because the influence and power of those states whose delegates are numerous will be *greater*, when compared to the influence and power of the other States, than the *proportion* which the numbers of their delegates bear to each other; as, for instance, though Delaware has *one* delegate, and Virginia but *ten*, yet Virginia has *more* than *ten times* as much *power* and *influence* in the government as Delaware.[1]

Martin evidently assumed that each state delegation in the House would cast its votes as a bloc, so he counted up various voting combinations of states in order to support his claim. Martin's objection

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[1] Cited by Riker (1986). Martin's report can be found in Farrand (1937), Vol. 3; the quotation appears on pp.198-199 with all emphasis in the original.

to apportioning seats proportionally to population correctly anticipated one of the fundamental propositions of modern voting power analysis — namely, that voting power may not be proportional to voting weight. This principle is most evident in the extreme case in which a single voter has a bare majority of the voting weight and therefore all the voting power, an example of which Martin provided earlier in his report (1937, p. 182).

Of course, Martin's expectation that state delegations in the House would cast bloc votes was not borne out. However, as noted at the outset, state electoral votes for President would soon be cast in blocs, and the U.S. Electoral College has subsequently been one of the principal institutions to which voting power analysis has been applied.

The mode of *apportioning* electoral votes is fixed in the Constitution: each state has electoral votes equal to its total representation in Congress, i.e., its House seats (apportioned on the basis of its population but with every state guaranteed at least one) plus two (for the Senators to which every state is entitled). Thus each state is guaranteed three electoral votes, and the apportionment reflects population only above this floor. The relative magnitude of the small-state advantage in electoral votes is determined by the ratio of the size of the Senate to the size of the House. While this ratio varied between about 0.19 and 0.29 during the nineteenth century, it has remained essentially constant at about 0.22 over the last hundred years.[2] Since 1912 the size of the House has been fixed at 435, since 1959 there have been 50 states, and since 1964 the 23rd Amendment has granted three electoral votes to the District of Columbia, so the total number of electoral votes at present is 538, with a bare majority of 270 votes required for election. Since 1964 a 269-269 electoral vote tie has been possible, so a Presidential election may be "thrown into the House of Representatives" (for lack of an electoral vote winner) even in the absence of third-party candidates winning electoral votes. In this event, the Constitution provides that the House of Representatives will choose between the tied candidates, with each state delegation casting one vote.

Additional Representatives (and electoral votes) beyond the floor of three are apportioned among the states on the basis of population. Since 1940, the "Hill-Huntington" apportionment formula (a divisor method also known as the "Method of Equal Proportions") has been used for this purpose, even though this method appears to have a slight small-state bias (Balinski and Young, 1982). Figure 1A shows the apportionment of House seats following the 2000 census in relation to the population of each state. Evidently approximate proportionality is achieved but, because apportionment must be in whole numbers, apportionment cannot be perfect. This *Whole Number Effect* is most conspicuous among small states, as is highlighted in Figure 1B. Figures 2A and 2B show the present apportionment of electoral votes in relation to population. It is evident that the small-state advantage resulting from the three electoral vote floor more than outweighs the capriciously unfavorable way some small states (in particular Montana, the largest state with only one House seat) are awarded House seats.

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[2] However, Congress has the power to change the size of the House without a constitutional amendment, so in principle it can increase or decrease the small-state advantage in electoral vote apportionment by decreasing or increasing the size of the House. For an analysis of how House size can influence the outcome of Presidential elections, see Neubauer and Zeitlin (2003).

The manner of selecting electors (and thereby the manner of casting electoral votes) is not fixed in the Constitution. Rather the Constitution empowers the legislature of each state to decide how to do this. In early years, the manner of selecting electors was subject to regular manipulation by politicians seeking state and (especially) party advantage (most notably in advance of the bitterly contested 1796 and 1800 elections). But since about 1836, with only few exceptions, electors in each state have been popularly elected on a "general ticket" and therefore have cast their electoral votes on the winner-take-all basis noted at the outset. Each party in each state nominates a slate of elector candidates, equal in number to the state's electoral votes and pledged to vote for the party's Presidential and Vice-Presidential candidates; voters vote for one or other slate (not individual electors) and the slate that wins the most votes is elected and casts its bloc of electoral votes as pledged. By standard voting power calculations (and as anticipated by Martin), this winner-take-all practice produces a large-state advantage that in some measure counterbalances the small-state advantage in the apportionment of electoral votes.

2. Banzhaf Voting Power

A measure of *a priori* voting power takes account of the fundamentals of a voting rule but nothing else. Thus the following analysis takes account only of the 2000 population of each state and the District of Columbia, the apportionment of electoral votes based on that population profile, and the requirement that a Presidential candidate receive 270 electoral votes to be elected. It does not take account of other demographic factors, historical voting patterns, differing turnout rates, relative party strength, survey or polling data, etc. This indicates the sense in which *a priori* voting power analysis is conducted behind a "veil of ignorance" and is blind to empirical contingencies.

In their authoritative treatise on the measurement of voting power, Felsenthal and Machover (1998) conclude that the appropriate measure of *a priori* voting power in typical voting situations, including the Electoral College, is the *absolute Banzhaf* (or *Penrose*) *measure* (Penrose, 1946; Banzhaf, 1968). Like other voting power measures, the Banzhaf setup is based on votes and outcomes that are both binary in nature, as in a two-candidate election. The Banzhaf measure is defined as follows.

Given n voters, there are 2^{n-1} *bipartitions* (i.e., complementary pairs of subsets) of voters (including the pair consisting of the set of all voters and the empty set). A voter (e.g., a state) is *critical* in a bipartition if the set to which the voter belongs is *winning* (e.g., a set of states controlling 270 electoral votes) but would not be winning if the voter belonged to the complementary set. A voter's *Banzhaf score* is the total number of bipartitions in which the voter is critical. A voter's *absolute Banzhaf voting power* is the voter's Banzhaf score divided by the number of bipartitions.

While this ratio may seem *ad hoc* and without theoretical justification, it has an intuitive and coherent rationale in terms of probability. If we know nothing about a voting situation other than its formal rules, our *a priori* expectation is that everyone votes randomly, i.e., as if independently flipping fair coins. In such a *random voting* (or *Bernoulli*) *model*, each bipartition of voters into complementary sets, in which everyone in one set votes for one candidate and everyone in the other set for the other candidate, has equal probability of occurring. Therefore, *a voter's absolute Banzhaf* *voting power is the probability that the voter's vote is decisive*, i.e., determines the outcome, in what we may call a *Bernoulli election*.

In a simple one-person, one-vote majority-rule system with an odd number n of voters, the *a priori* voting power of a voter is the probability that the election is otherwise tied. If the number of voters n is even, the voter's voting power is one half the probability that the vote is otherwise within one vote of a tie. Provided that n is greater than about 25, this probability is very well approximated by the expression $\sqrt{2/\pi n}$. This expression implies that, in a simple majority rule situation, *individual voting power* is inversely proportional, not to the number of voters, but to the *square root of the number of voters*. We refer to the $\sqrt{2/\pi n}$ formula as the *Inverse Square Root Rule* for simple majority rule voting. Given simple-majority rule *Bernoulli elections* with n voters, the expected vote for each candidate is 50%, the probability that each candidate wins is .5, and the standard deviation of either candidate's absolute vote (over repeated elections) is approximately $.5\sqrt{n}$.

Calculating Banzhaf voting power values in voting situations in which voters have unequal weights is considerably more burdensome. Direct enumeration by the Banzhaf formula informally sketched out above is feasible (even using a computer) only if the number of voters does not exceed about 25. It is possible to make exact calculations (using a computer) for up to about 200 voters by using so-called generating functions. Dennis and Robert Leech have created a website for making voting power calculations using these and other methods.[3] I have used this website, together with the Inverse Square Root Rule, to make all the direct calculations reported below.

3. Voting Power in the Existing Electoral College

The *a priori* Banzhaf voting power of states in the current Electoral College is shown in Table 1. (Since equal electoral votes imply equal voting power, states need not be individually listed.) For the moment, I ignore the fact that Maine and Nebraska actually award their electoral votes in the manner of the "Modified District System" discussed below. Remember that a state's Banzhaf voting power is the probability that its block of electoral votes is decisive in a Bernoulli election. Thus, with 55 electoral votes, California's Banzhaf voting power of about .475 means that, if we repeatedly flip fair coins to determine how each state other than California casts its electoral votes, about 47.5% of the time neither candidate would have 270 electoral votes before California casts its votes and therefore either (i) California's bloc of 55 votes would determine the winner or (ii) the leading candidate would have exactly 269 electoral votes and California's 55 votes would either elect that leading candidate or create a tie. (In the latter event, the Banzhaf measure in effect awards California "half credit.")

Figure 3 shows each state's share of voting power in relation to its share of electoral votes. Only California has a noticeably larger share of voting power than of electoral votes and, even for this mega-state, voting power only slightly exceeds voting weight. This is a manifestation of the *Penrose Limit Theorem*, which states that voting power tends to become proportional to voting

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[3] *Computer Algorithms for Voting Power Analysis* (http://www.warwick.ac.uk/~ecaae/). The calculations in this paper used its *ipgenf* algorithm.

weight as the number of voters increases, provided that the distribution of voting weights is not "too unequal."[4]

Figure 4 shows each state's share of voting power in relation to its share of the U.S. population. The small-state apportionment advantage still shows up quite prominently, and even California's noticeable advantage with respect to voting power does not fully compensate for its disadvantage with respect to apportionment (though California does better than all intermediate-sized states).

But the 51-state Electoral College weighted voting system depicted in Figure 3 is largely a chimera, since states are not voters but merely geographical units within which popular votes are aggregated. A U.S. Presidential election really is a *two-tier* voting system, in which the casting of electoral votes is determined by the popular vote within each state. So we now turn to the power of individual voters under the Electoral College system.

One distinct advantage of the absolute Banzhaf power measure is its applicability to two-tier voting systems such as the Electoral College. The voting power of an individual voter depends on both his voting power in the simple majority election within the voter's state and the voting power of that state in the Electoral College itself. Since both voting power values can be interpreted as probabilities, they can be multiplied together to get the voter's overall two-tier voting power. That is to say, the *a priori* voting power of an individual voter in the Electoral College system (as it works in practice) is the probability that the voter casts a decisive vote in his state (given by the Inverse Square Root Rule) multiplied by the probability that the bloc of votes cast by the voter's state is decisive in the Electoral College (given by the calculations displayed in Table 1) or, as we may say informally, the probability of "double decisiveness."

Putting the Inverse Square Root Rule and the Penrose Limit Theorem together (and referring to the units in the second tier generically as "districts"), we can derive the following expectations pertaining to two-tier voting systems. First, if the voting weight of districts is proportional to the number of voters in each, individual two-tier voting power *increases* proportionately with the square root of the number of voters in the voter's district. We call this the *Banzhaf Effect*. Second, if the voting weight of districts is equal (regardless of the number of voters in each), individual two-tier voting power *decreases* proportionately with the square root of the population of the voter's district. We call this the *Inverse Banzhaf Effect*. Given the preceding considerations, we can anticipate the approximate results of Banzhaf calculations of individual two-tier voting power under the Electoral College to be as follows.

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[4] This "theorem" is actually a conjecture that has been proved in important special cases and supported by a wider range of simulations; see Lindner and Machover (2007) and Chang et al. (2006). The number of voters need not be very large in order for the theorem statement to be true to good approximation. Indeed, given the provisional apportionment of 65 House seats among only thirteen states that was the focus of Luther Martin's objections, the Penrose Limit Theorem held to reasonable approximation (Virginia's advantage then was roughly comparable to California's today), so Martin's complaint about the disproportionate voting power of large states, while theoretically insightful, was in the circumstances largely off-the-mark (but also see footnote 8).

(1) Individual voting power within each state is inversely proportional to the square root of the number of voters in the state (due to the Inverse Square Root Rule).

(2) As shown in Chart 3, state voting power in the Electoral College is approximately proportional to its voting weight, i.e., its number of electoral votes (due to the Penrose Limit Theorem).

(3) As shown in Chart 2, the voting weight of states in turn is approximately (apart from the small-state apportionment advantage) proportional to population (and therefore to the number of voters).

(4) As shown in Chart 4 and putting (2) and (3) together, state voting power is approximately proportional to population.

(5) So putting together (1) and (4), individual *a priori* two-tier voting power is approximately proportional to the square root of the number of voters in a state. However, this large-state advantage is counterbalanced in some degree by the small-state apportionment advantage.

In his pioneering analysis of voting power in the Electoral College (based on the 1960 Census), Banzhaf (1968) reached the following conclusion.

> [A] voter in New York State has 3.312 times the voting power of a citizen in another part of the country. . . . Such a disparity in favor of the citizens of New York and other large states also repudiates the often voiced view that the inequalities in the present system favor the residents of the less populous states.[5]

Table 2 shows how *a priori* individual voting power under the existing Electoral College (based on the 2000 Census) varies across selected states. The calculations that underlie this table and the subsequent charts assume that the number of voters in each state is a fixed percent of its population in the 2000 Census.[6] The last column shows individual two-tier voting power rescaled in the manner suggested in the Banzhaf quotation above, i.e., so that the individual two-tier voting power of voters in the least favored state (Montana, the largest state with only three electoral votes) is 1.0000 and other power values are multiples of this. The last row shows individual voting power, under the same assumptions about electorate size, given direct (single-tier) popular election of the President.

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[5] Specifically, Banzhaf found that voters in New York (the largest state at the time of the 1960 census) had 3.312 times the voting power of voters in the District of Columbia; they had 2.973 times the voting power of voters in the least favored state (Maine). The maximum disparity resulted from the stipulation in the 23rd Amendment that the District cannot have more electoral votes than the least populous state. In the 1960 census, the District not only had a population larger than every state with 3 electoral votes but also larger than several states with 4 electoral votes. The District today has a smaller population than every state except Wyoming.

[6] This was taken to be 43.37%, which is equal to the total popular vote for President in 2004 (122,294,000) as a percent of the U.S. population in 2000. *A priori*, we have no reason to expect that the percent of the population that is eligible to vote, or of eligible voters who actually do vote, varies by state (though, empirically and *a posteriori*, we know there is considerable variation in both respects). Using a different (fixed) percent of the population to determine the number of voters in each state would (slightly) affect the following estimates of absolute individual voting power but not comparisons across states or Electoral College variants.

Figure 5 shows rescaled individual two-tier voting power of voters in all 50 states plus the District of Columbia, the mean voting power of all voters in the existing Electoral College, and individual (single-tier) voting power given a direct popular election. Note that the latter is substantially greater than mean individual voting power under the Electoral College — indeed, it is greater than individual voting power in every state except California. So by the criterion of maximizing individual *a priori* voting power (which is hardly the only relevant criterion), only voters in California would have reason to object to replacing the Electoral College by a direct popular vote. On average, individual voting power would be about 1.35 times greater given a direct popular vote than under the existing Electoral College.

We noted earlier that a 269-269 electoral vote tie is a possibility. The preceding Banzhaf voting power calculations take account of the possibility of such a tie, but they do not take account of what happens in the event of such a tie as consequence of other provisions in the U.S. Constitution. What happens is that the election is "thrown into the House of Representatives," whose members choose between the two tied candidates, but with the very important proviso that each state delegation casts one vote. If we assume that voters in each state in effect vote for a slate of House members (just as they vote a slate of electors), individual voting power in this contingent runoff election is equal to what it would be if electoral votes were apportioned equally among the states (as discussed in the next section and depicted in Figure 9). A fully comprehensive assessment of individual voting power under the existing Electoral College would take account of the probability of such a contingent indirect election. The one additional calculation needed is the probability that the Electoral College produces a 269-269 vote tie in a Bernoulli election, which is approximately .007708.[7] Clearly the effect of taking account of this contingent procedure in which states have equal weight is to reduce the power of voters in large states and increase the power of voters in small states. However, the District of Columbia does not participate in this contingent election (since it has no Representative), so the power of voters in DC is reduced by the probability of an electoral vote tie. Only the power of California voters is noticeably (but not greatly) reduced, while the power of voters in states with up to about 7 electoral votes is noticeably (but not greatly) increased, when we take account of House runoffs.

More realistically, second-tier voting by state delegations in the event of an Electoral College tie is not (like states voting in the Electoral College) a chimera. Since Representatives would not have been elected by first-tier voters on the basis of their prospective vote for President, and state delegations would typically be internally divided, the more realistic conception is that, in the event of an electoral vote tie, ordinary voters drop out of the picture and individual House members vote in the first tier, their votes are aggregated within state delegations, and each state cast its single "electoral vote" accordingly in the second tier. From this more realistic point of view, the small-state advantage in the second-tier is substantially diluted by the Inverse Banzhaf Effect within delegations. For example, the second-tier voting power of each state delegation is .112275; the single Delaware Representative is always decisive within his delegation and thus has individual two-tier voting power of .112275; each member of the 15-member Michigan delegation has first-tier voting power .209473

[7] This probability can be derived from other values calculated and displayed by the *ipgenf* algorithm of *Computer Algorithms for Voting Power Analysis*.

and therefore individual two-tier voting power of .023519 — a bit over $^1/_5$ (not $^1/_{15}$) that of the Delaware member.[8]

4. Voting Power and the Apportionment of Electoral Votes

We next consider two types of variants of the existing Electoral College system. Variants of the first type retain the winner-take-all practice for casting electoral votes but employ different formulas for apportioning electoral votes among states. Variants of the second type retain the existing apportionment of electoral votes among the states but change the winner-take-all practice for casting electoral votes (or, in one case, adds "national" electoral votes).

Variants of the first type include the following:

(a) apportion electoral votes in whole numbers entirely on the basis of population (e.g., on the basis of House seats only);
(b) apportion electoral votes fractionally to be precisely proportional to population;
(c) apportion electoral votes fractionally to be precisely proportional to population but then add back the two electoral votes based on Senate representation; and
(d) apportion electoral votes equally among the states (in the manner of House voting on tied candidates), so that the winning candidate is the one who carries the most states.

Figure 6 shows individual voting power with electoral votes apportioned by House seats only. At first blush it may not look much different from Figure 5, but it is important to take careful note of the vertical scale. Removing the small-state apportionment advantage in this way has the consequence of making voting power in the most favored state of California about ten times (rather than about three times) greater than that in Montana. Since apportionment is still in whole numbers, capricious inequalities remain among states with quite similar small populations.

Figure 7 shows individual voting power with electoral votes apportioned precisely by population (so states have fractional electoral votes). Since we have removed both the small-state apportionment advantage and the capricious effects of apportionment into whole numbers, the Banzhaf Effect as it pertains to individual first-tier voting power is essentially all that matters, and individual voting power increases smoothly and almost perfectly with the square root of state population. Voting power in California remains about ten times greater than voting power in the least favored state; however that least favored state is now Wyoming, as it is the smallest state and therefore has the least (fractional) electoral vote weight.

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[8] Had Luther Martin's concern been the two-tier voting power of *individual members* of the House (rather than the voting power of *state delegations*) under the assumption of bloc voting by state delegations, his complaint that states should not have representation proportional to population would have been strongly supported by the theory of voting power measurement, because large-state members benefit from the (direct) Banzhaf Effect. Using the same example, under the Martin setup the Delaware and Michigan delegations have second-tier voting power of .008314 and .125606 respectively, so their members have two-tier voting power of .008314 and .026311 respectively, the latter being more than three times greater than the former. (The *relative* voting power of House members under the Martin setup would be essentially that displayed in Figure 6.)

Figure 8 shows individual voting power with House electoral votes apportioned precisely by population but with the two Senatorial electoral votes added back in. Individual voting power continues to vary smoothly with population but in a non-monotonic fashion, as the relationship takes on a hockey-stick shape. The voting power advantage of California voters falls back again to about three times that of the least favored state. As the population of a voter's state increases, the small-state apportionment advantage diminishes but at a declining rate, while the large-state voting power advantage due to Banzhaf Effect increases. Idaho happens to be at the point on the population scale where these effects balance out, and it is therefore the least favored state.

Figure 9 shows individual voting power when all states have equal voting weight and is in a sense the inverse of Figure 7. Since all states have equal second-tier voting power, individual voting power varies only with respect to first-tier voting power, and therefore smoothly reflects the Inverse Banzhaf Effect.

We may ask whether it is possible to apportion electoral votes among the states so that, even while retaining the winner-take-all practice, individual two-tier voting power is equalized across all states. One obvious but constitutionally impermissible possibility is to redraw state boundaries so that all states have the same number of voters (and electoral votes). Equalizing state populations in this way to create a system of *uniform apportionment* that not only equalizes individual voting power across states, but also increases mean individual voting power, relative to that under any type of apportionment based on actual and unequal state populations (Kolpin, 2003). However, even while increased as well as equalized, individual voting power still falls below the (equal) individual voting power under direct popular vote. So the fact that mean individual voting power under the Electoral College falls below that under direct popular vote is not due mainly to the fact that states are unequal in population and cast their unequal electoral votes on a winner-take-all basis; rather it is evidently intrinsic to any strictly two-tier system.

Given that state boundaries are immutable, can we apportion electoral votes so that (without changing state populations and preserving the winner-take-all practice) the voting power of individuals is equalized across all states? Individual voting power can be equalized (to a high degree of perfection) by apportioning electoral votes so that state voting power is proportional to the square root of state population. This entails using what Felsenthal and Machover (1998, p. 66) call the *Penrose Square Root Rule*. Such *Penrose apportionment* can be tricky, because what must be made proportional to population is not electoral votes (which is what we directly apportion) but state voting power (which is a consequence of the whole profile of electoral vote apportionment). However, in the case of the Electoral College we can immediately come up with an excellent approximation, which can be refined as desired. This is because, as we saw earlier, each state's share of voting power in the Electoral College is close to its voting weight, because $n = 51$ is large enough, and the distribution of state populations is equal enough, for the Penrose Limit Theorem to hold to very good approximation. So simply apportioning electoral votes to be precisely proportional (by allowing fractional electoral weight) to the square root of the population (or number of voters) in each state, we achieve almost perfect equality of voting power (call this *pure Penrose apportionment*); further refinement seems unnecessary, especially as electoral votes probably must be apportioned into whole numbers anyway. But even if we must be content with *whole-number Penrose apportionment*, we can make individual voting power much more equal than it is now or would be under any of the Electoral College variants examined here, other than the National Bonus Plan with a large bonus. Once again the whole-number effect capriciously advantages or disadvantages small states much more than large states, as is shown in Figure 10. The chart once again makes clear that equalizing individual voting power is not the same as maximizing it (as under direct popular vote).

5. Voting Power and the Casting of Electoral Votes

We now consider the second type of variant of the existing Electoral College system. These variants retain the existing apportionment of electoral votes but employ rules other than winner-take-all for the casting of electoral votes.[9] Such variants include the following.

The Pure District Plan. Each state is divided into as many equally populated single-member districts as it has electors, and one elector is elected from each district. In effect, each party ticket earns one electoral vote for each district it wins.[10]

The Modified District Plan. Electors apportioned to a state on the basis of House seats are elected from the same equally populated Congressional Districts as the House members; the two additional electors apportioned to each state on the basis of their two Senate seats are (like the Senators) elected at-large. In effect, a party ticket earns one electoral vote for each Congressional District it wins and two electoral votes for each state it wins.[11]

The Pure Proportional System. The electoral votes of each state are cast (fractionally) for party tickets in precise proportion to their state popular vote totals.[12]

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[9] Many of these Electoral College variants have actually been proposed as constitutional amendments, while few if any amendments have proposed changes in the apportionment of electoral votes. For a review of proposed constitutional amendments pertaining to the Electoral College, see Peirce and Longley (1981), especially Chapter 6 and Appendix L. However, provided the position of Presidential elector is retained, each state legislature is free to change its mode of casting electoral votes (or, more directly, its mode of selecting Presidential electors) and, as previously noted, Maine and Nebraska actually depart from the winner-take-all arrangement at the present time.

[10] Evidently most members of the Constitutional Convention expected that electors would be popularly elected in this manner. However, their Constitution left this matter up to individual states legislatures. Under the original Electoral College system, each elector cast two undifferentiated votes for President. The candidate with the most votes became President (provided he received votes from a majority of electors) and the runner-up became Vice-President. After the first two contested Presidential elections in 1796 and 1800, it was clear that this system could not accommodate elections in which two parties each ran a ticket with both a Presidential and Vice-Presidential nominee. Following the election of 1800, there was considerable consensus to change the manner of casting electoral votes so that each elector would cast one designated vote for President and one designated vote for Vice President, and this was accomplished by the Twelfth Amendment. Though early drafts included the requirement that electors be popularly elected in the manner of the Pure District Plan, this provision was ultimately dropped from the amendment; see Kuroda (1994).

[11] This is the system used at present by Maine (since 1972) and Nebraska (since 1992). The 2008 election for the first time produced a split electoral vote in Nebraska, where Obama carried one Congressional District; the Republican-dominated legislature may now switch state law back to winner-take-all. A proposed constitutional amendment (the Mundt-Coudert Plan) in the 1950s would have mandated the Modified District Plan for all states.

[12] A proposed constitutional amendment (the Lodge-Gossett Plan) along these lines was seriously considered in Congress in the late 1940s and 1950s. Since fractional electoral votes would be cast, the position of Presidential elector would necessarily be abolished, so this change can be effected only by constitutional amendment. Since minor

The Whole Number Proportional Plan. The electoral votes of each state are cast in whole numbers for party tickets on the basis of an apportionment formula applied to the state popular vote.[13]

The National Bonus Plan. Existing electoral votes are apportioned and cast as at present, but the national popular vote winner earns an additional electoral vote bonus of some magnitude.[14]

Voting power calculations for the Pure District Plan can be made in just the same way as our previous results. Calculations for the other plans require somewhat different modes of analysis. In particular, those for the National Bonus and Modified District Plans present formidable difficulties, because each voter casts a single vote that counts in two ways.

Under the Pure District Plan, all voters in the same state have equal first-tier voting power, which can be calculated by the Inverse Square Root Rule, with n equal to the number of voters in the state divided by its number of electoral votes. Since the second-tier voting is also unweighted, the second-tier voting power of each district can also be calculated by the Inverse Square Root Rule with $n = 538$. Figure 11 shows individual voting power by state under the Pure District Plan. Inequalities come about entirely because of apportionment effects — in particular, the small-state apportionment advantage and the whole-number effect. The small-state advantage in apportionment carries through to voting power — for example, voters in Wyoming have almost twice the voting power as those in California, but it is substantially diluted by the Inverse Banzhaf Effect. While California districts have almost four times as many voters as the Wyoming districts, California voters have about half the voting power of those in Wyoming.

Under the Modified District Plan, two electors are elected at-large in each state and the others are elected by Congressional Districts. Individual voting power within each state is equal, because each district has an equal number of voters. All districts have equal voting power in the Electoral College, because they have equal weight, i.e., 1 electoral vote; and all states have equal voting power in the Electoral College, because they have equal weight, i.e., 2 electoral votes. But individual voting

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candidates would presumably win (fractional) electoral votes, it becomes more likely that neither major candidate would win a majority of the electoral votes, so such an amendment would also have to specify what would happen in this event. (The Lodge-Gossett Plan would have elected the electoral-vote plurality winner, unless that candidate failed to receive at least 40% of the electoral votes, in which case Congress voting by joint ballot would choose between the top two candidates ranked by electoral votes.)

[13] Since electoral votes would still be cast in whole numbers, the position of elector can be retained, and a state may use this formula unilaterally. Indeed, such a system was proposed in Colorado as initiative Proposition 36 in 2004. Since third candidates would be likely to win a few electoral votes (especially in large states), this system, if widely adopted, would throw more elections into the House.

[14] The principal purpose of such a plan is evidently to reduce the probability of an "election reversal" of the sort that occurred in 2000. The larger the national bonus, the more this probability is reduced. A bonus of 102 electoral votes has been most commonly discussed. (It would make sense, however, to make the bonus an odd number so as to preclude electoral vote ties.)

power across states is not equal, because districts in different states have different numbers of voters (due to the whole-number effect) and states with different populations have equal electoral votes.

The Modified District Plan is more complicated than it may at first appear, as the same votes are aggregated in two different ways, with the result that doubly decisive votes can be cast in three distinct contingencies: (i) a vote is decisive in the voter's district (and the district's one electoral vote is decisive in the Electoral College); (ii) a vote is decisive in the voter's state (and the state's two electoral votes are decisive in the Electoral College); and (iii) a vote is decisive in both the voter's district and state (and the combined three electoral votes are decisive in the Electoral College).

Moreover, because each individual vote counts in two ways, there are interdependencies in the way in which district and state electoral votes may be cast. Whichever candidate wins the two statewide electoral votes must win at least one district electoral vote as well but need not win more than one. Thus in a state with a single House seat, individual voting power under the Modified District Plan operates in just the same way as under the existing Electoral College, as its three electoral votes are always cast in a winner-take-all manner for the state popular vote winner. In a state with two House seats, the state popular vote winner is guaranteed a majority of the state's electoral votes (i.e., either 3 or 4) and a 2-2 split cannot occur. In a state with three or more House seats, electoral votes may be split in any fashion and, in a state with five or more House seats, the statewide popular vote winner may win only a minority of the state's electoral votes — that is, "election reversals" may occur at the state, as well as the national, level.

However, the preceding remarks pertain only to logical possibilities. Probabilistically, the casting of district and statewide electoral votes will to some degree be aligned in Bernoulli (and other) elections. Given that a candidate wins a given district, the probability that the candidate also wins statewide is greater than 0.5 — that is to say, even though individual voters cast statistically independent votes, the fact that they are casting individual votes that count in the same way in two tiers (districts and states) induces a correlation between popular votes at the district and state levels within the same state. This correlation, which is perfect in the states with only one House seat, diminishes as a state's number of House seats increases, and therefore enhances individual voting power in small states relative to what it is under the Pure District Plan. But this correlation also makes the calculation of individual two-tier voting power far from straightforward.

The first step is to determine the probability of each of the three first-tier contingencies in which a voter may be doubly decisive. (See Miller, 2008, for further details.) The probability that the district vote is tied can be calculated by the Inverse Square Root Rule, and likewise the probability that the statewide vote is tied. The conditional probability that the state vote is tied, given that the district vote is tied, is equal to the probability that the popular vote cast in all other districts in the state together is tied, which can be calculated by the Inverse Square Root Rule. By multiplying this conditional probability by the probability that the district vote is tied in the first place, we get the probability that both district and state votes are tied, i.e., the probability of contingency (iii) above. The probabilities of the two other contingencies can then be determined by simple subtraction.

Having determined the probability of each contingency that makes a voter decisive in the first tier, we must calculate the probability that the single electoral vote of the district, or the two electoral votes of the state, or the combined three electoral votes of both (as the case may be) are decisive in the second tier. At first blush, it might seem that we need only evaluate the voting power of units within an Electoral College of 436 units with one electoral vote each and 51 units with two electoral votes each, but to do this ignores the interdependencies and correlations discussed earlier.[15]

While it may be possible to proceed analytically, I have found the obstacles to be formidable and have instead proceeded by generating a sample of 120,000 Bernoulli elections, with electoral votes awarded to the candidates on the basis of the Modified District Plan.[16] This generated a database that can be manipulated to determine frequency distributions of electoral votes for the focal candidate under specified contingencies with respect to first-tier voting, from which relevant second-tier probabilities can be inferred.[17]

Figure 12 shows individual voting power across the states under the Modified District Plan. Voters in small states are more favored than under the Pure District Plan (because small states come closer to maintaining winner-take-all than larger states), but the Inverse Banzhaf effect within each state still attenuates the small-state apportionment advantage relative to their advantage under the Pure Proportional Plan, to which we now turn.

With sufficiently refined proportionality, the Pure Proportional Plan creates a 122-million *single-tier* (rather than a two-tier) *weighted* voting system, where the weight of individual votes is given by their state's electoral votes divided by the number of voters in the state.[18] The calculations

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[15] Banzhaf (1968, pp. 320 and 331) presented calculations for the Modified District Plan that ignored these interdependencies. Had he displayed the *absolute* voting power of voters in each state, it would have been evident that mean individual voting power under the district plan (as he calculated it) exceeded that under direct popular vote, which Felsenthal and Machover (1998, pp. 58-59) show is an impossibility. However, his *rescaled* voting power values are quite close to those presented here.

[16] The simulation took place at the level of the 436 districts, not individual voters. For each Bernoulli election, the popular vote for the focal candidate was generated in each Congressional District by drawing a random number from a normal distribution with a mean of $n/2$ and a standard deviation of $.5\sqrt{n}$ (i.e., the normal approximation of the symmetric Bernoulli distribution), where n is the number of voters in the district. (Of course, the other candidate won the residual vote.) The winner in each district was determined, the district votes in each state were added up to determine the state winner, and electoral votes are allocated accordingly.

[17] Even with the very large sample, few elections were tied at the district or state level, so the relevant electoral vote distributions were taken from a somewhat wider band of elections, namely those that fell within 0.2 standard deviations of an exact tie. (In a standard normal distribution, the ordinate at $\pm 0.2\times$SDs from the mean is about .98 times that at the mean.) It needs to be acknowledged that Figure 12 (and Figures 15A and 15B for the National Bonus Plan) are not as accurate as other figures, since they entail some sampling error, some other approximations (including the one just noted), and possibly other errors.

[18] The Lodge-Gossett Plan proposed in the 1950s specified that candidates would be credited with fractional electoral votes to the nearest one-thousandth of an electoral vote. As proportionality becomes less refined, this system begins to resemble the Whole-Number Proportional System. The Proportional Plan has recently been reinvented as the "Weighted Vote Shares" proposal of Barnett and Kaplan (2007). Combining a precisely proportional method of *casting* of electoral votes with a precisely proportional *apportionment* of electoral votes (as discussed earlier) would give every voter equal weight and would be equivalent to direct popular vote.

displayed in Figure 13 assume that proportionality is sufficiently refined to create a single-tier weighted voting system and use the Penrose Limit Theorem to justify the assumption that voting power is proportional to voting weight in this very large-n single-tier weighted voting system.[19] It can be seen that under this plan the small-state apportionment advantage carries through to individual voting power without the dilution evident under the Pure District Plan (or, to a lesser degree, under the Modified District Plan), because in a single-tier voting system there is no room for the Inverse Banzhaf Effect. Thus the fact that Wyoming has almost four times the electoral votes per capita as California translates without dilution into voting power for Wyoming voters that is likewise almost four times that of California votes. Finally, since sufficiently refined proportionality creates what is effectively a weighted single-tier voting game (with relatively equal weights), mean individual voting power is essentially equal to (but in principle slightly less than) individual voting power under a direct (unweighted) popular vote.[20]

The Whole-Number Proportional Plan divides a state's electoral votes between (or among) the candidates in a way that is as close to proportional to the candidates' state popular vote shares as possible, given that the apportionment must be in whole numbers. In principle, there are as many such plans as there are apportionment formulas. In addition (and as under many proportional representation electoral systems), candidates might be required to meet some vote threshold in order to win electoral votes.[21] But, in the event there are just two candidates (as we assume here), all apportionment formulas work in the same straightforward way: multiply each candidate's share of the popular vote by the state's number of electoral votes to derive his electoral vote *quota* and then round this quota to the nearest whole number in the normal manner.[22] In this two-tier system, individual *a priori* voting power is the probability that the voter casts a decisive vote within his or her state, in the sense that other votes in the state are so divided that the individual's vote determines whether a candidate gets k or $k+1$ electoral votes from the state and that this single electoral vote is decisive in the Electoral College (where, as usual, these probabilities result from Bernoulli elections).

Figure 14 shows that the Whole-Number Proportional Plan produces a truly bizarre allocation of voting power among voters in different states.[23] Voters in the seventeen states with an even

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[19] Banzhaf (1968, pp. 319 and 330) presented similar calculations based on similar, though less explicit, assumptions.

[20] Mean voting power under the Pure Proportional Plan (as calculated here) is .000072150172 versus .0000721502396 under direct popular vote.

[21] Colorado's Proposition 36 had no explicit vote threshold but used a distinctly *ad hoc* apportionment formula that was overtly biased in favor of the leading candidate and against minor candidates.

[22] Given three or more candidates, simple rounding does not always work, because the rounded quotas may not add up to the required number of electoral votes — hence the "apportionment problem" definitively treated by Balinski and Young (1982).

[23] Similar calculations and chart were independently produced and have since been published by Beisbart and Bovens (2008).

number of electoral votes are rendered (essentially) powerless. Voters in the 33 states and the District of Columbia with an odd number of electoral votes have voting power (essentially) as if each of these states had equal voting weight (in the manner of Figure 9). Here's why this happens.

In a Bernoulli election with fairly large number of voters, the vote essentially always is divided almost equally between the two candidates. As previously noted, the expected vote share for each candidate is .5 with a standard deviation of $.5\sqrt{n}$. Consider a state with four electoral votes. For its electoral votes to be divided otherwise than 2 to 2, one candidate must receive more than 62.5% of the vote, because 0.625×4 earns an electoral vote quota of 2.5 electoral votes, and anything below this rounds to 2. Such a state has about 500,000 voters, so the expected vote share for either candidate in a Bernoulli election is 250,000 with a standard deviation of about $.5\sqrt{500{,}000} = 354$ votes. Since a candidate has to receive 62,500 votes (about 175 standard deviations) above this expected vote share in order for anyone to cast a decisive vote, it is essentially guaranteed that the electoral vote will be split $2-2$, giving each voter essentially zero probability of casting a decisive vote. As the even number of electoral votes increases, two things change. First, the relative vote margin required to produce anything other than an even split of electoral votes decreases. For example, in a state with 50 electoral votes, a candidate needs to get only 51% of the vote to earn a quota over 25.5 electoral votes, anything above which rounds off to 26. At the same time, while the absolute standard deviation of the expected vote percent increases with the square root of electorate size, the relative standard deviation (expressed as a percent of the vote) decreases with the square root of electorate size. Overall, the gap between the required margin and 50% relative to the standard deviation diminishes with electorate size, but not nearly fast enough to give voters measurable *a priori* voting power in even the largest states.

With respect to the 34 states with an odd number of electoral votes, the results are only slightly less bizarre. For (appropriate) example, consider Colorado with 9 electoral votes. Whichever candidate receives the most popular votes wins at least 5 electoral votes. But to win more than 5 electoral votes, a candidate must earn an electoral vote quota of more than 5.5 (rounding to 6), which requires a bit over 61% of the popular vote. Even in state with 55 electoral votes (e.g., California), one candidate must win a bit over 51.8% of the votes to win more than 28 of them. By the same considerations that applied in the even electoral vote case, the probability of achieving such margins in a Bernoulli election is essentially zero. Thus in each state with an odd number of electoral votes, effectively only one electoral vote is at stake, and the distribution of voting power is effectively the same as if electoral votes were equally apportioned among these states, thereby giving a huge advantage to voters in smaller states with an odd number of electoral votes.

Finally, we take up the National Bonus Plan, focusing particularly on a bonus of 101 electoral votes. In this event, there are 639 electoral votes altogether, with 320 required for election (and ties are precluded). As with the Modified District Plan, doubly decisive votes can be cast in three distinct contingencies: (i) a vote is decisive in the voter's state (and the state's electoral votes are decisive in the Electoral College); (ii) a vote is decisive in the national election (and the national bonus is decisive in the Electoral College); and (iii) a vote is decisive in both the voter's state and in the national election (and the combined state and bonus electoral votes are decisive in the Electoral College).

The probabilities of the first-tier contingencies can be calculated in the same manner as those for the Modified District Plan. I then generated a sample of 256,000 Bernoulli elections, with electoral votes awarded to the candidates on the basis of the National Bonus Plan (with bonuses of varying magnitudes). Again this generated a database that can be manipulated to determine frequency distributions of electoral votes for the focal candidate under specified contingencies with respect to first-tier voting, from which relevant second-tier probabilities can be inferred.

Figure 15A displays individual voting power with a national bonus of 101 electoral votes. At first blush, Figure 15A may look very similar to Figure 5 for the existing Electoral College, but inspection of the vertical axis reveals that the inequalities between voters in large and small states are greatly compressed relative to the existing system. Figure 15B displays individual voting power under national bonuses of varying magnitude. A bonus of zero is equivalent to the existing Electoral College system and a bonus of 533 is logically equivalent to direct popular vote,[24] though Figure 15B indicates that any bonus greater than about 150 is essentially equivalent to direct popular vote. Sampling error presumably accounts for the minor anomalies in this chart, but the overall patterns are clear enough. As the size of the bonus increases, voting power inequalities are compressed and mean individual voting power increases until it equals that under direct popular vote.

6. *Conclusions*

I conclude with a few summary points, observations, and qualifications.

Figure 16 summarizes and compares individual two-tier voting power under all Electoral College variants that entail unequal voting power. In this chart, voting power must be expressed in absolute terms, rather than be rescaled so that the voting power of the least favored voter is 1.00, because it makes comparisons across Electoral College variants under which different voters are least favored and the absolute voting power of the least favored voters varies. While the existing Electoral College favors voters in large states with respect to *a priori* voting power, all alternative electoral vote-casting plans would shift the balance of voting power quite dramatically in favor of voters in small states. The National Bonus Plan is a partial exception, in that it reduces the large-state advantage as the magnitude of the bonus increases and equalizes voting power given a sufficiently large bonus.

The ten columns of plotted points in Figure 16 indicate that there are substantial differences among the plans with respect to both the mean level of individual voting power and inequality of voting power. The first point is highlighted in Figure 17A, which ranks all variants (now including uniform and Penrose apportionments, plus direct popular election) with respect to the mean level of individual voting power that they entail. Direct popular election establishes a maximum that cannot be exceeded, but it is essentially matched by the Pure Proportional Plan and the National Bonus Plan (with a bonus of 101) does almost as well. The Modified District Plan follows some distance behind.

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[24] Just as a statewide winner under the Modified District Plan must win at least one district, the national popular vote winner must win at least one state and 3 electoral votes; 533 is the smallest number B such $B + 3 > 538 - 3$.

At the lower extreme, the Whole-Number Proportional Plan, which renders a large proportion of voters powerless, ranks well below all other variants, while the remaining variants are all clustered quite closely together in the middle of the range.

Figures 17B and 17C focus on inequality of individual voting power. Figure 17B summarizes information that is also directly apparent in Figure 16, by ranking the Electoral College variants with respect to the ratio of maximum to minimum individual voting power that they entail. This ratio is essentially infinite under the Whole-Number Proportional Plan (favoring small states with an odd number of electoral votes), and it is very high (favoring large states) when electoral votes are apportioned (whether in fractions or whole numbers) proportional to population and it is also high (but favoring small states) when states have equal electoral weights. Direct popular vote and uniform apportionment achieve perfect equality, as does pure Penrose apportionment if sufficiently refined. Whole-number Penrose apportionment does almost as well. The remaining systems are clustered fairly close together in the lower middle portion of the range. Figure 17C assesses the variants with respect to inequality of voting power more comprehensively in terms of the ratio of the standard deviation to the mean of individual voting power. The same five variants achieve perfect or close-to-perfect equality, and the Whole-Number Proportional Plan remains the extreme outlier in the other direction, though it can now be placed at a definite point on the scale. The other systems are ranked much as in Figure 17B but are spread over a larger portion of the total range.

The analysis presented in this paper has been static, in particular by considering Electoral College variants in turn and assuming that the manner in which states cast their electoral votes is fixed and uniform. But states are free to switch unilaterally from the existing winner-take-all system to either district plan or to the Whole-Number Proportional Plan. Therefore, it is worth observing that, in so far as states chose their mode of casting electoral votes with an eye to maximizing the power of their voters, the existing (almost) universal winner-take-all method is an "equilibrium choice"— that is, no state (or small subset of states) has an incentive to switch from winner-take-all to one of the available alternatives. For example, in the mid-1990s the Florida state legislature gave serious consideration to a proposal to use the Modified District Plan, though it ultimately rejected the proposal. The effect on individual voting power of a switch by Florida away from winner-take-all is shown in Figure 18A (which, however, assumes a switch to the Pure District Plan, because the calculations are straightforward). Individual voting power in Florida would have been cut to about one sixth of its previous magnitude, while the power of voters in all other states would have been slightly increased.[25] Likewise, had Colorado voters passed Proposition 36 and put the Whole-

[25] It would appear that Maine and Nebraska have been penalizing themselves in the same fashion for several decades, but the penalty for departing from winner-take-all is much less severe for smaller states. If Maine used the Pure District System instead of winner-take-all, the power of its voters would be cut approximately in half. Since it actually uses the Modified District Plan and is small enough that this system entails "winner-take-almost-all" (i.e., at least three of its four electoral votes), the actual reduction in voting power of Maine voters is less than this. (Another consequence of a Florida switch to districts would have been that — at least considering "mechanical" effects only — Gore would have been elected President in 2000, with no room for dispute and regardless of who won the statewide vote in Florida.)

Number Proportional System into effect in their state, they would have (with respect to *a priori* voting power) in effect been throwing away four of their five electoral votes — or all of them, in the event Colorado were to gain (or lose) a House seat in the next apportionment.

Moreover, a universal winner-take-all system is not simply an equilibrium choice; it appears to be the only equilibrium, and it has strongly "attractive" as well as "retentive" properties. For example, prior to the 1800 election, Massachusetts switched from a mixed system of selecting Presidential electors to legislative appointment, which in practice meant winner-take-all for the locally dominant Federalist Party. A concerned Jefferson wrote to Monroe (cited by Pierce and Longley, 1981, p. 37):

> All agree that an election by districts would be best if it could be general, but while ten states choose either by their legislatures or by a general ticket [i.e., in either event, winner-take-all], it is folly or worse for the other six not to follow.

At the instigation of Jefferson and the locally dominant Republican Party, Virginia switched from the Pure District Plan to winner-take-all a general ticket for the 1800 election. If it had not done so, the Jeffersonian Republicans might easily have lost enough Virginia districts to lose the national electoral vote. Figure 18B, though using the present apportionment of electoral votes, powerfully confirms Jefferson's strategic insight in terms of individual voting power (though the voting-power rationale for winner-take-all is logically distinct from Jefferson's party-advantage rationale). Given a universal district (or whole-number proportional) system, Massachusetts (or any other state, but large states even more than small states) would gain substantially by switching from districts to winner-take-all. As other states follow, they also would gain but not as much as Massachusetts initially did and they would erode the initial advantage of the earlier switchers. No equilibrium is reached until all states switch to winner-take-all, even though small states would end up worse off than at the outset. Moreover, at least under the present apportionment, mean voting power would end up slightly lower than at the outset. Even if a district system were universally agreed to be socially superior (as Jefferson evidently considered it), states are caught in a kind of Prisoner's Dilemma and would not voluntarily retain (or return to) such a system, though they would ratify a constitutional amendment mandating it nationwide.

Finally, I should acknowledge that there are several important critiques of Banzhaf voting power measurement as applied to the Electoral College and similar two-tier voting systems (e.g., Margolis, 1983; Gelman et al., 2002, 2004; Katz et al., 2004). These critiques rest fundamentally on the (indisputable) observation that Bernoulli elections are in no way representative of empirical voting patterns. But these critiques overlook the fact that the Banzhaf measure pertains to *a priori* voting power. It measures the power of states — and, in the two-tier version, of individual voters — in a way that takes account of the Electoral College voting rules *but nothing else*. As we have seen, a voter in California is about three times more likely to cast a decisive vote than one in New Hampshire in a Bernoulli election. But if we take account of recent voting patterns, poll results, and other information, a voter in New Hampshire undoubtedly has had greater *empirical* (or *a posteriori*) probability of decisiveness in recent elections, and accordingly got more attention from the candidates and party organizations than one in California. But if California and New Hampshire were both perfectly contested “battleground” states, California's *a priori* advantage would be surely reflected in its *a posteriori* voting power as well.

If it is hardly related to empirical voting power in any particular election, the question arises of whether *a priori* voting power should be of concern to political science and practice. I think the answer is yes. In particular, constitution-makers arguably should, and to some extent must, design political institutions from behind a “veil of ignorance” concerning empirical contingencies and future political trends. Accordingly they should, and to some extent must, be concerned with how the institutions they are designing allocate *a priori* voting power.

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EV	N	ABSOLUTE BANZHAF
3	8	.022730
4	5	.030312
5	5	.037900
6	3	.045493
7	4	.053094
8	2	.060704
9	3	.068324
10	4	.075955
11	4	.083599
12	1	.091257
13	1	.098930
15	3	.114328
17	1	.129805
20	1	.153194
21	2	.161043
27	1	.208805
31	1	.241422
34	1	.266331
55	1	.475036
538	51	4.166201

EV — Number of Electoral Votes
N — Number of States
ABSOLUTE BANZHAF — Absolute Banzhaf voting power
Calculated by *ipgenf* at *http://www.warwick.ac/~ecaae/*

TABLE 1

***A PRIORI* STATE VOTING POWER IN THE CURRENT ELECTORAL COLLEGE**

STATE	ELECT SIZE	IND VP	EV	STATE VP	IND 2-T VP	REL VP
MT	392640	.00127334	3	.022730	.00002894	1.000000
UT	970074	.00081010	5	.037900	.00003070	1.060803
DE	340488	.00136738	3	.022730	.00003108	1.073857
NH	537107	.00108870	4	.030312	.00003300	1.140203
OK	1500107	.00065145	7	.053094	.00003459	1.195039
AK	272771	.00152771	3	.022730	.00003472	1.199770
WS	2329521	.00052277	10	.075955	.00003971	1.371895
CO	1870085	.00058346	9	.068324	.00003986	1.377338
MD	2302057	.00052587	10	.075955	.00003994	1.380054
MA	2756442	.00048058	12	.091257	.00004386	1.515269
NC	3498990	.00042655	15	.114328	.00004877	1.684919
MI	4317893	.00038398	17	.129805	.00004984	1.722080
OH	4933195	.00035923	20	.153194	.00005503	1.901409
IL	5394875	.00034352	21	.161043	.00005532	1.911389
PA	5334862	.00034544	21	.161043	.00005563	1.922110
FL	6951810	.00030262	27	.208805	.00006319	2.183181
NY	8242552	.00027791	31	.241422	.00006709	2.318163
TX	9066167	.00026499	34	.266331	.00007057	2.438416
CA	14715957	.00020799	55	.475036	.00009880	3.413738
US	122294000	.00007215	538	—	.00007215	2.492845

ELECT SIZE — Size of Electorate
[2000 Population x .4337, where .4337 = 2004 Total Presidential Vote/2000 US Population]
IND VP — Individual Absolute Banzhaf Voting Power within State
[by the Inverse Square Root Rule]
STATE VP — State Absolute Banzhaf Voting Power (from Table 1)
IND 2-T VP — Individual Banzhaf Voting Power in Two-Tier System [= IND VP x ST VP]
REL VP — Relative Individual 2-T Voting Power (rescaled so that minimum [Montana] = 1)

TABLE 2

***A PRIORI* INDIVIDUAL VOTING POWER IN SELECTED STATES**

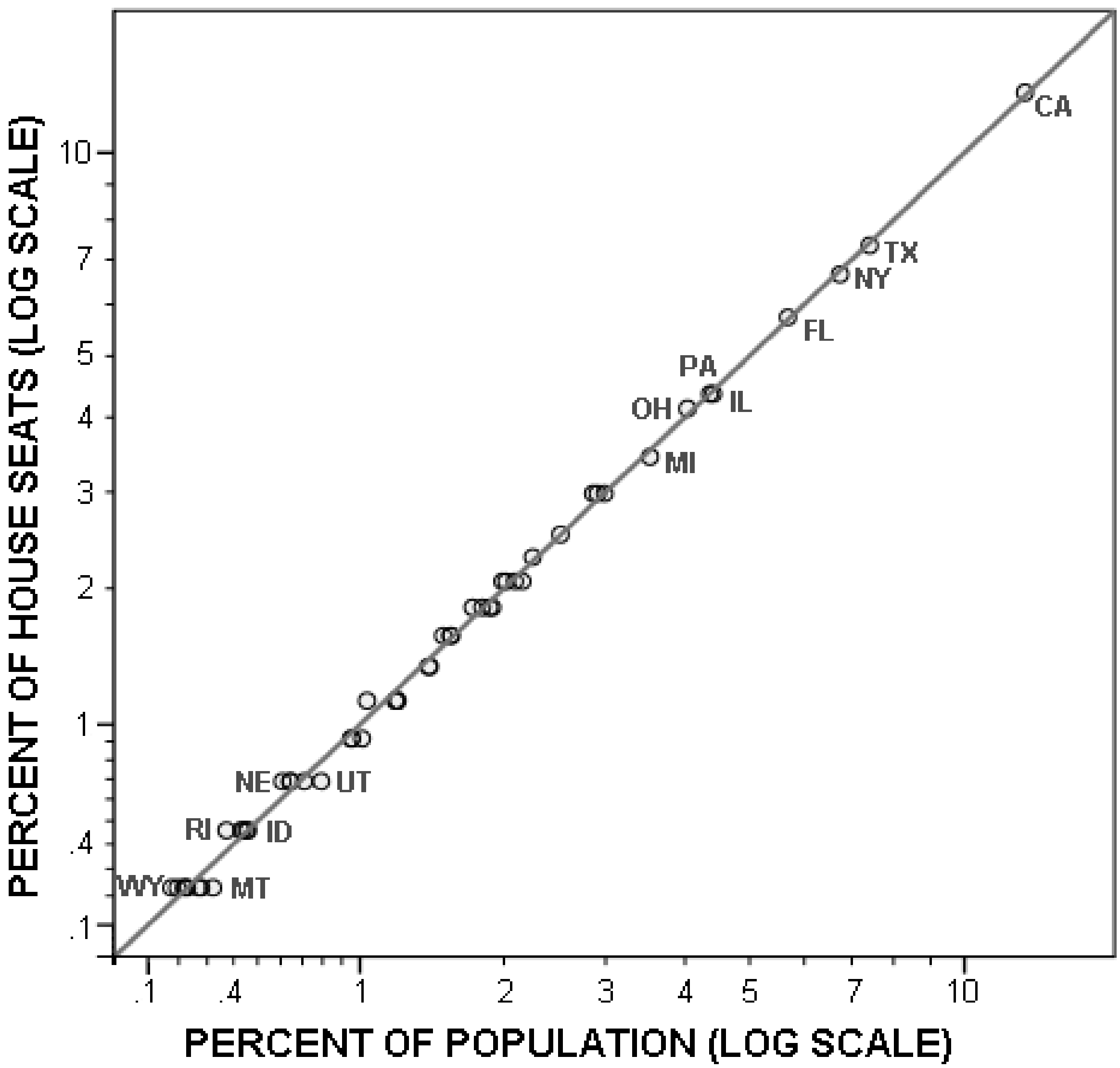


Figure 1A Apportionment of House Seats (2000 Census) by State Population

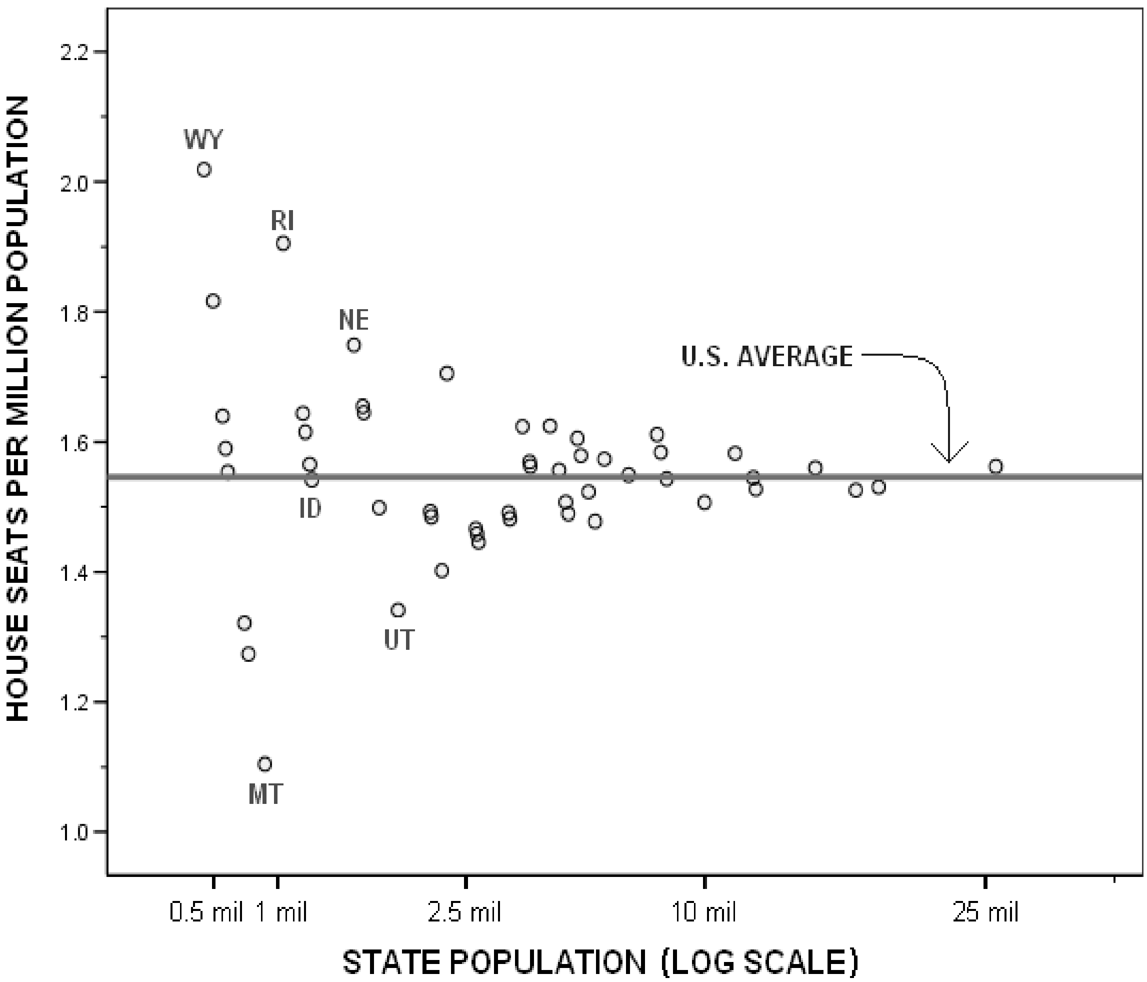
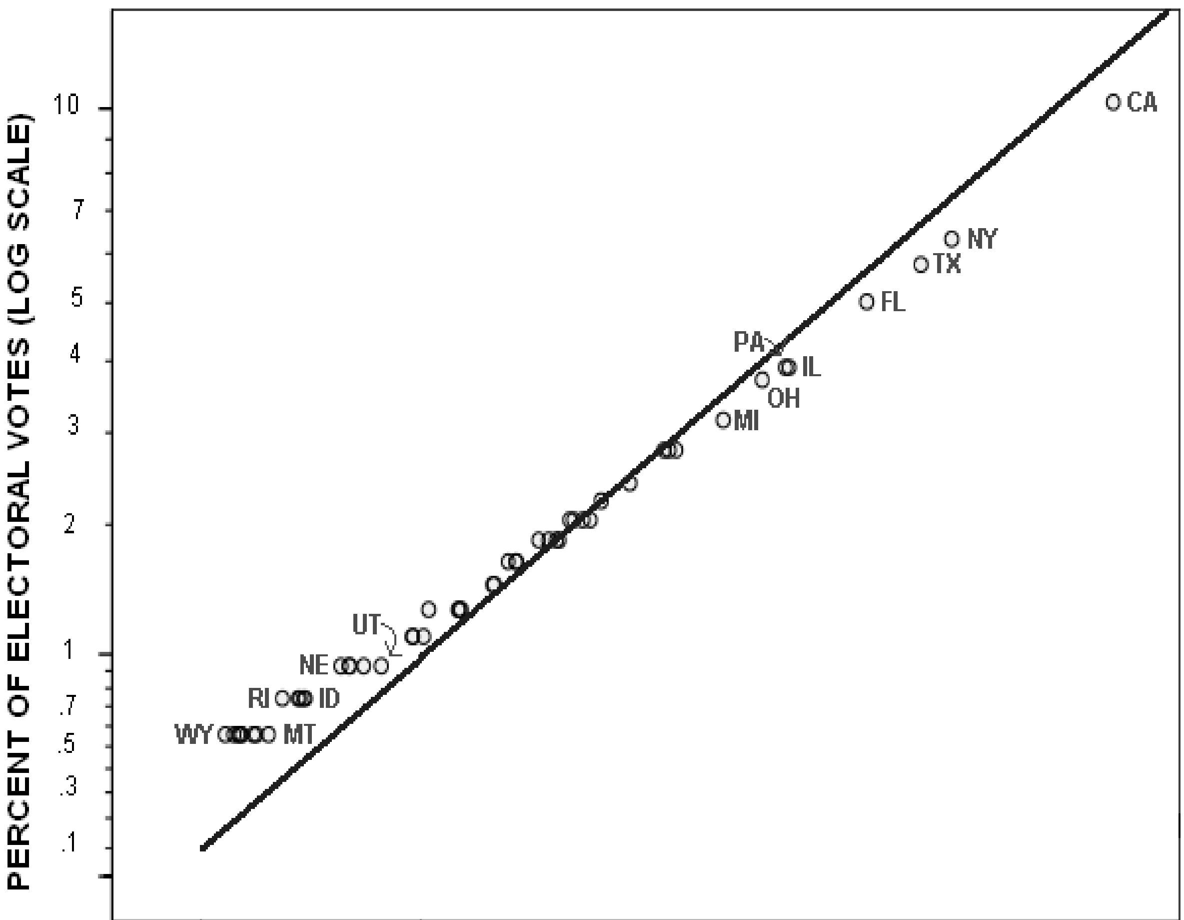


Figure 1B House seats per Million by State Population



PERCENT OF 2000 APPORTIONMENT POPULATION (LOG SCALE)

Figure 2A Apportionment of Electoral Votes by State Population

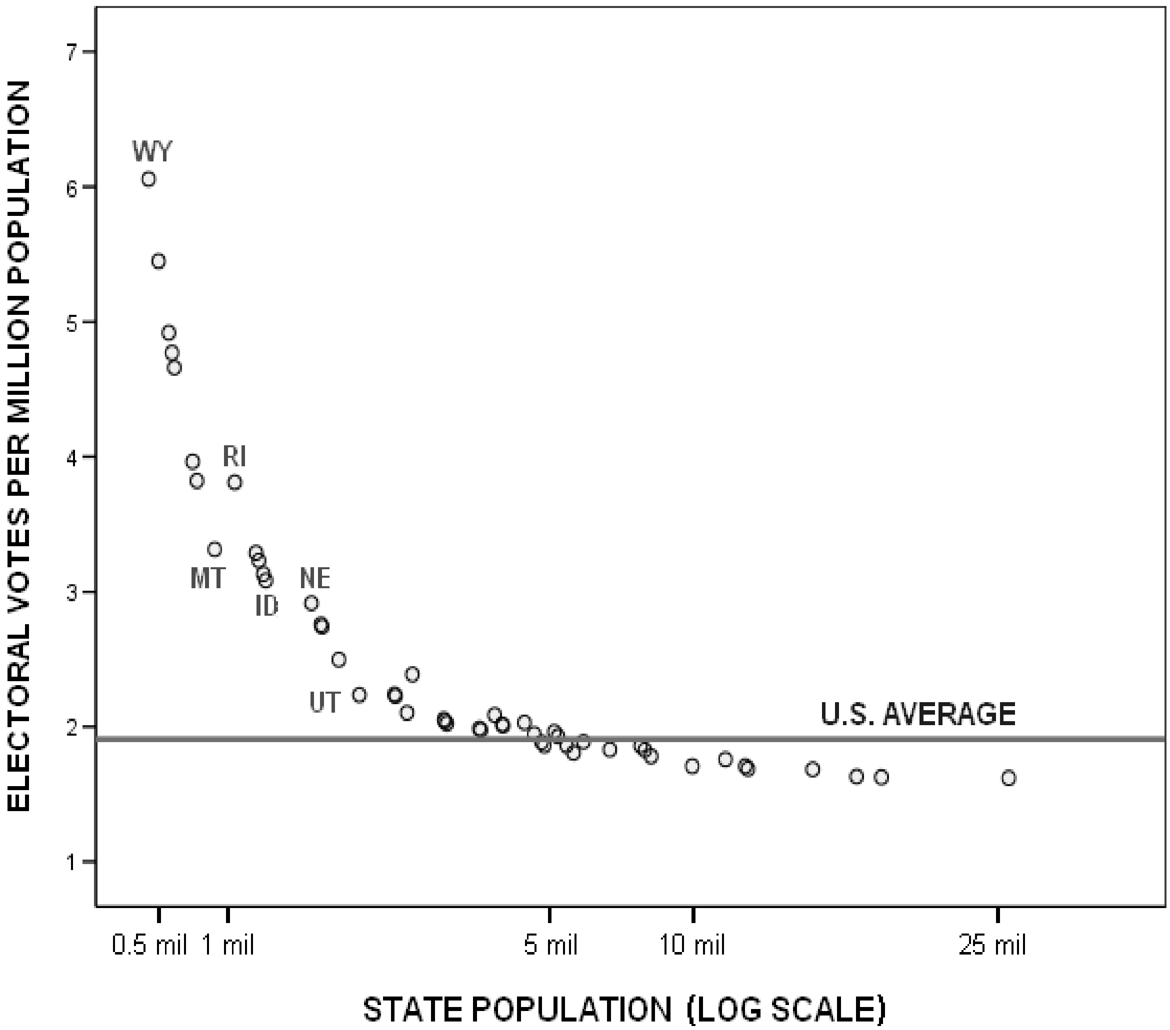


Figure 2B Electoral Votes Per Million by State Population

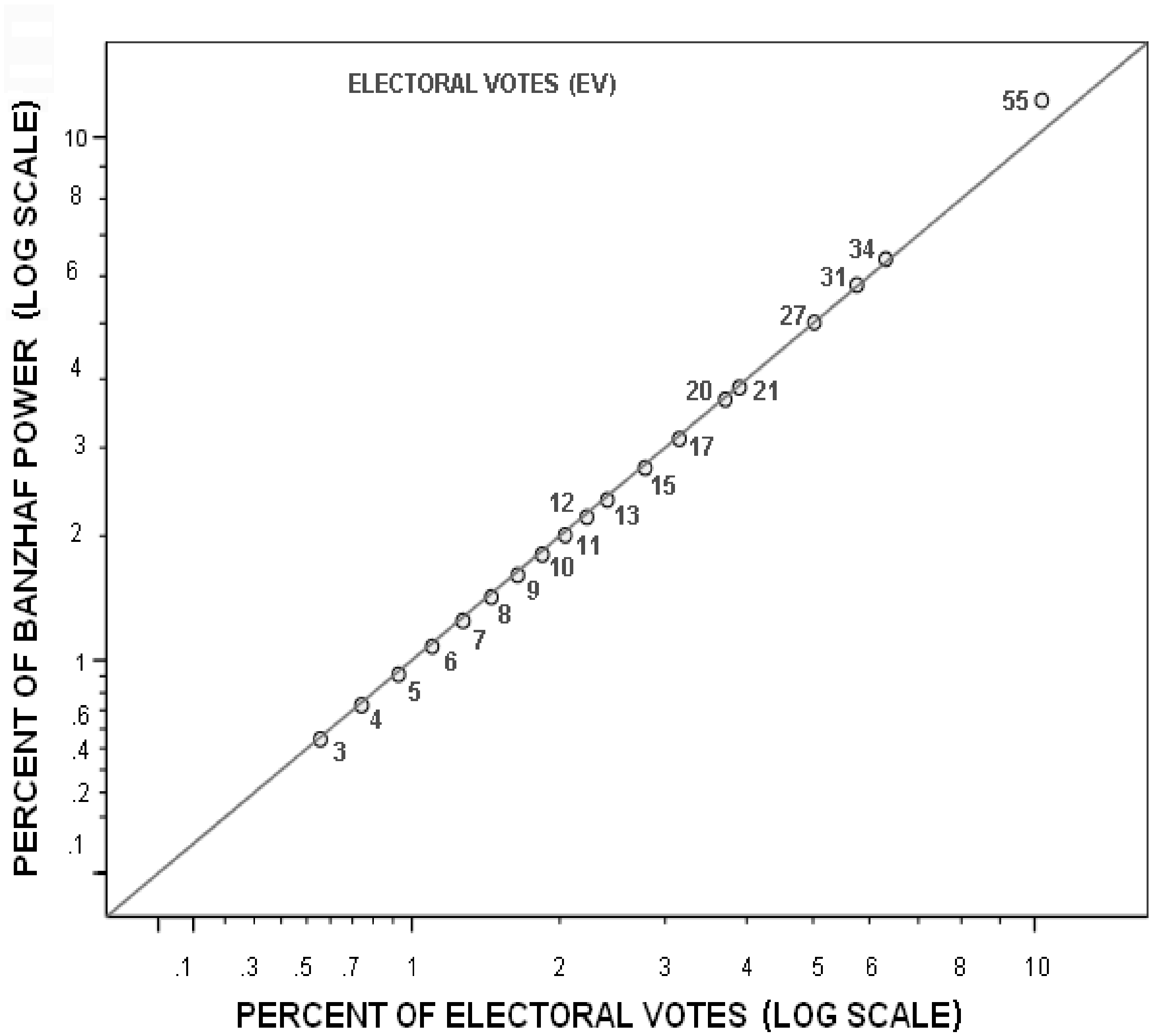


Figure 3 Banzhaf Voting Power of States by Electoral Votes

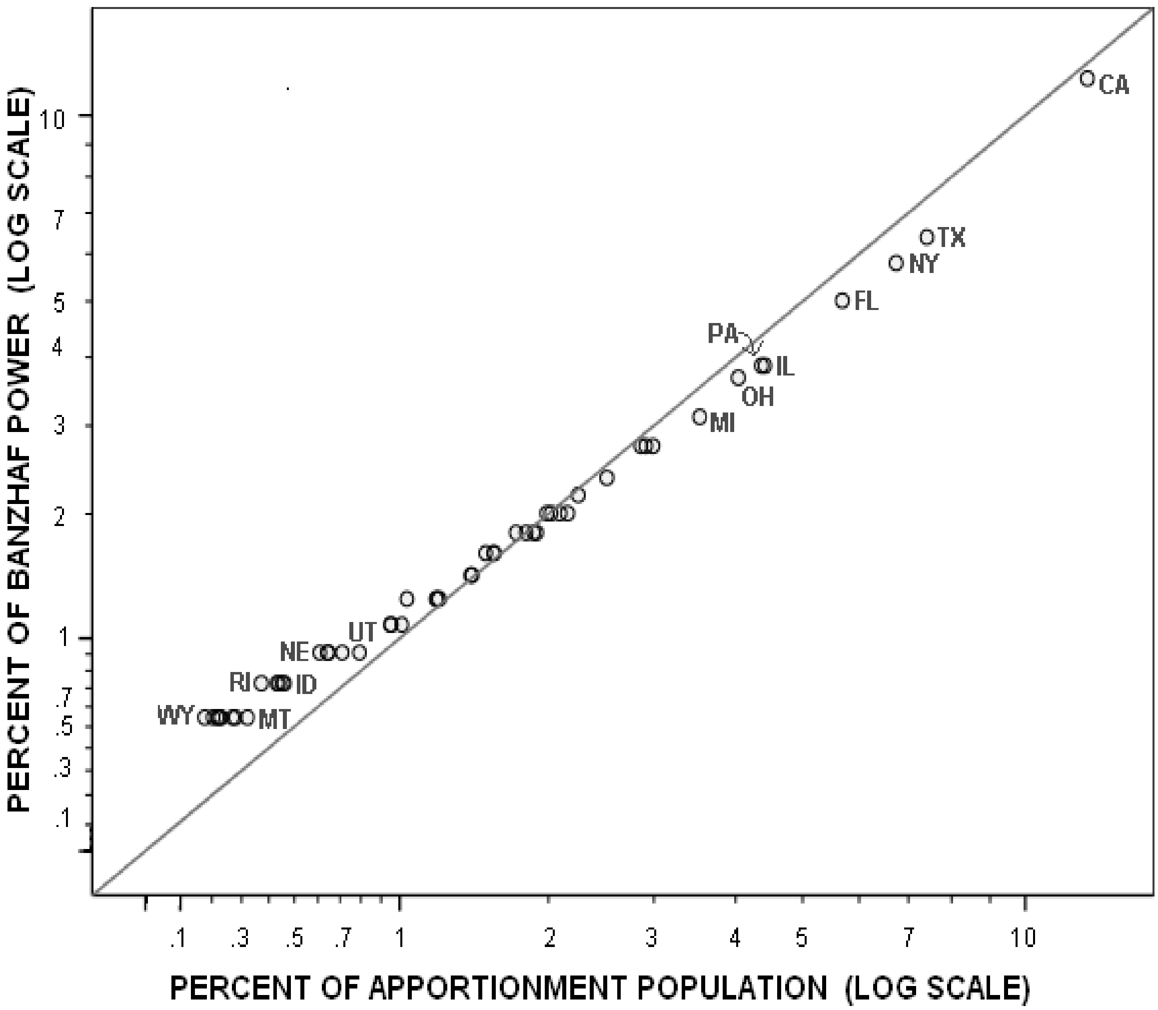


Figure 4 Banzhaf Voting Power of States by Population

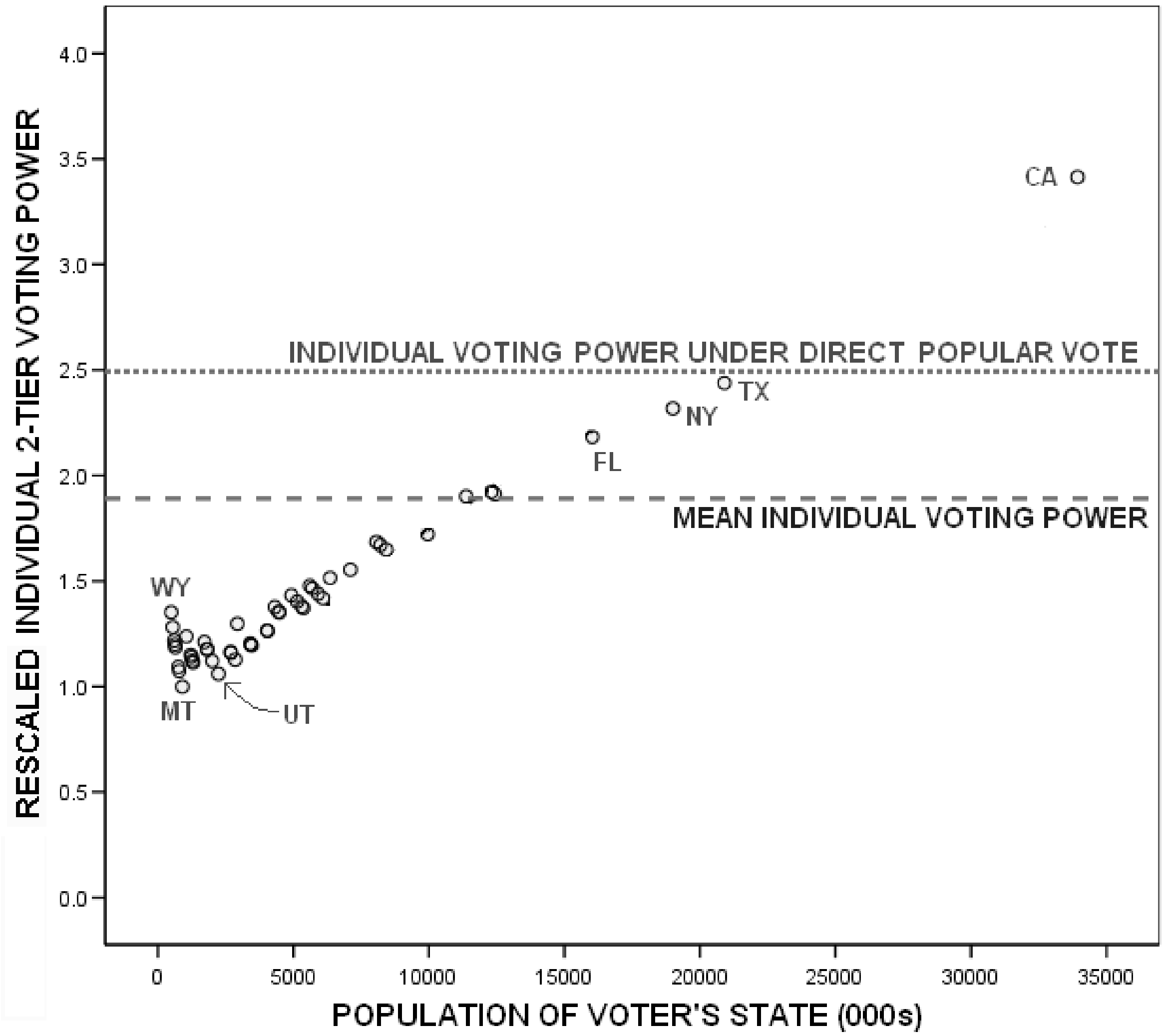


Figure 5 Individual Voting Power by State Population under the Existing Apportionment of Electoral Votes

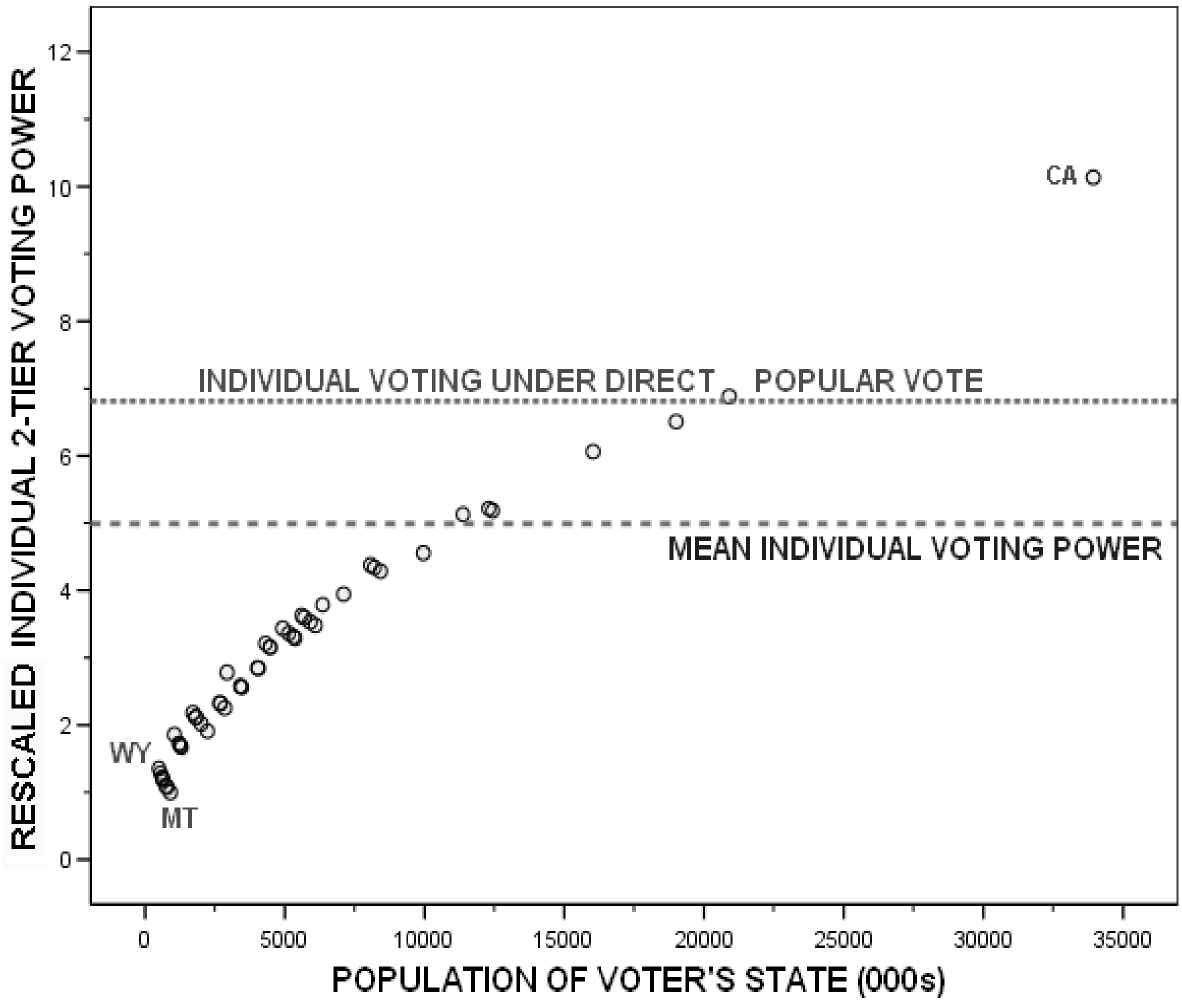


Figure 6 Individual Voting Power by State Population with Electoral Votes Based on House Seats Only

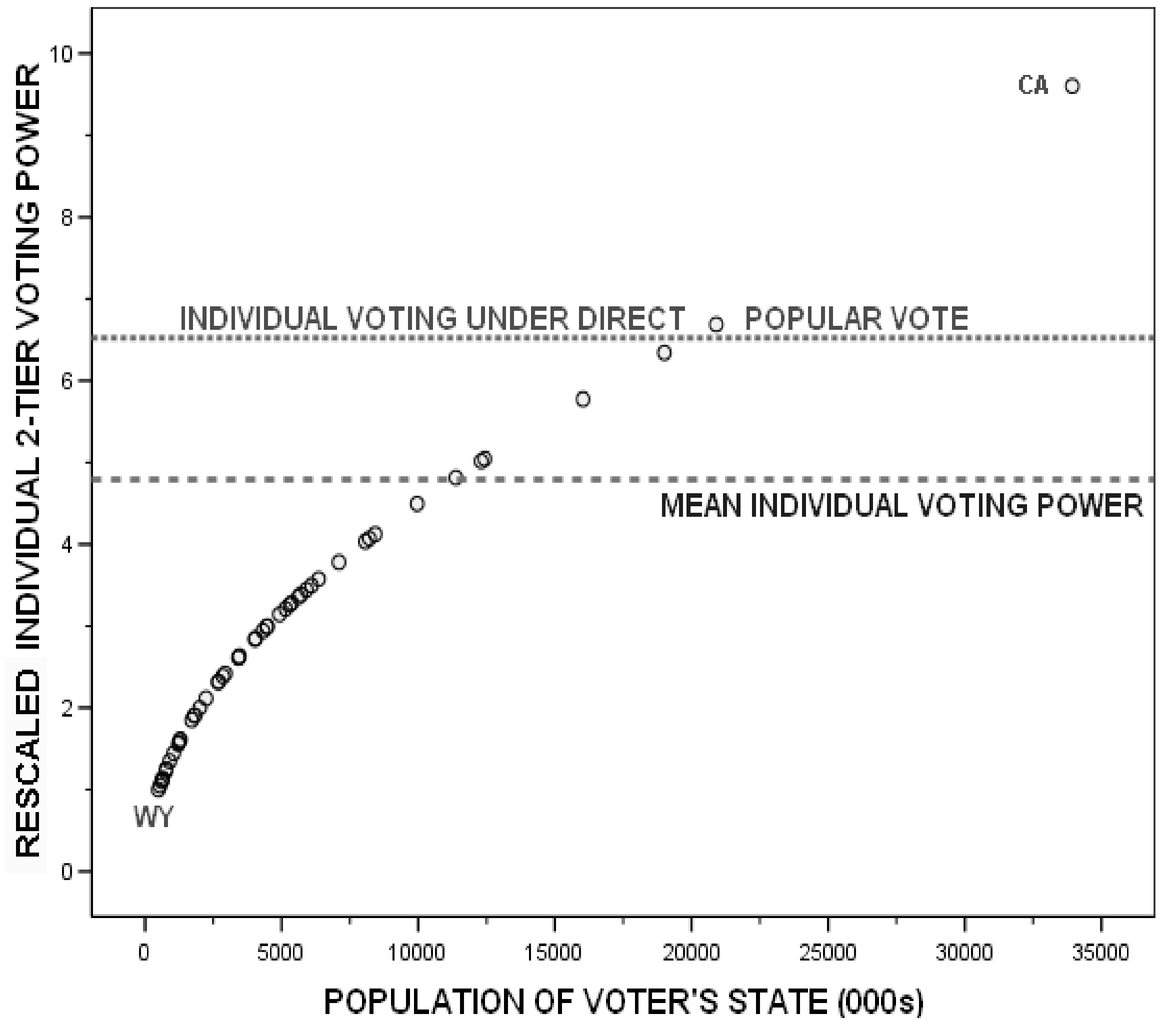


Figure 7 Individual Voting Power by State Population with Electoral Votes Precisely Proportional to Population

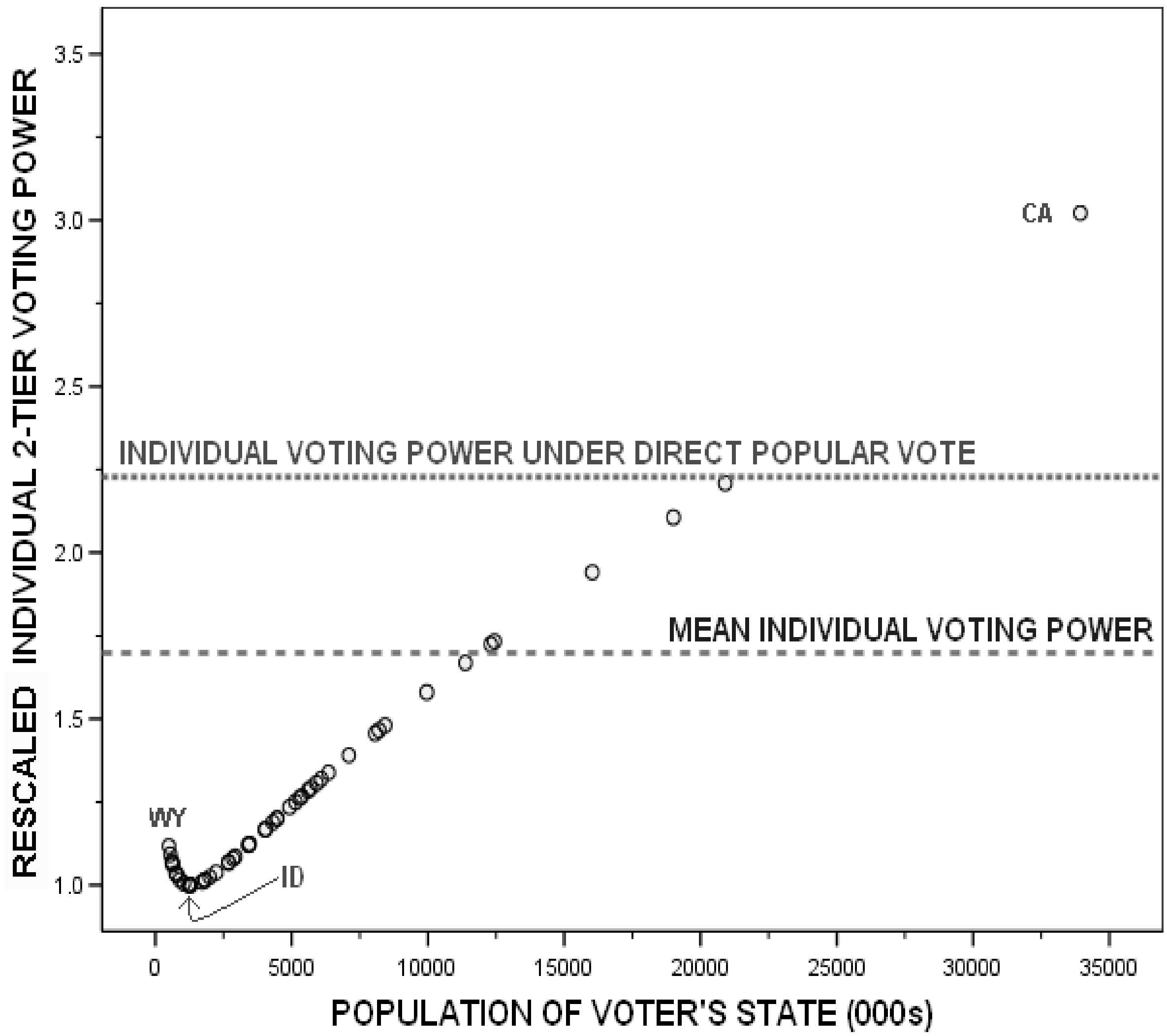


Figure 8 Individual Power by State Population with Electoral Votes Precisely Proportional to Population Plus Two

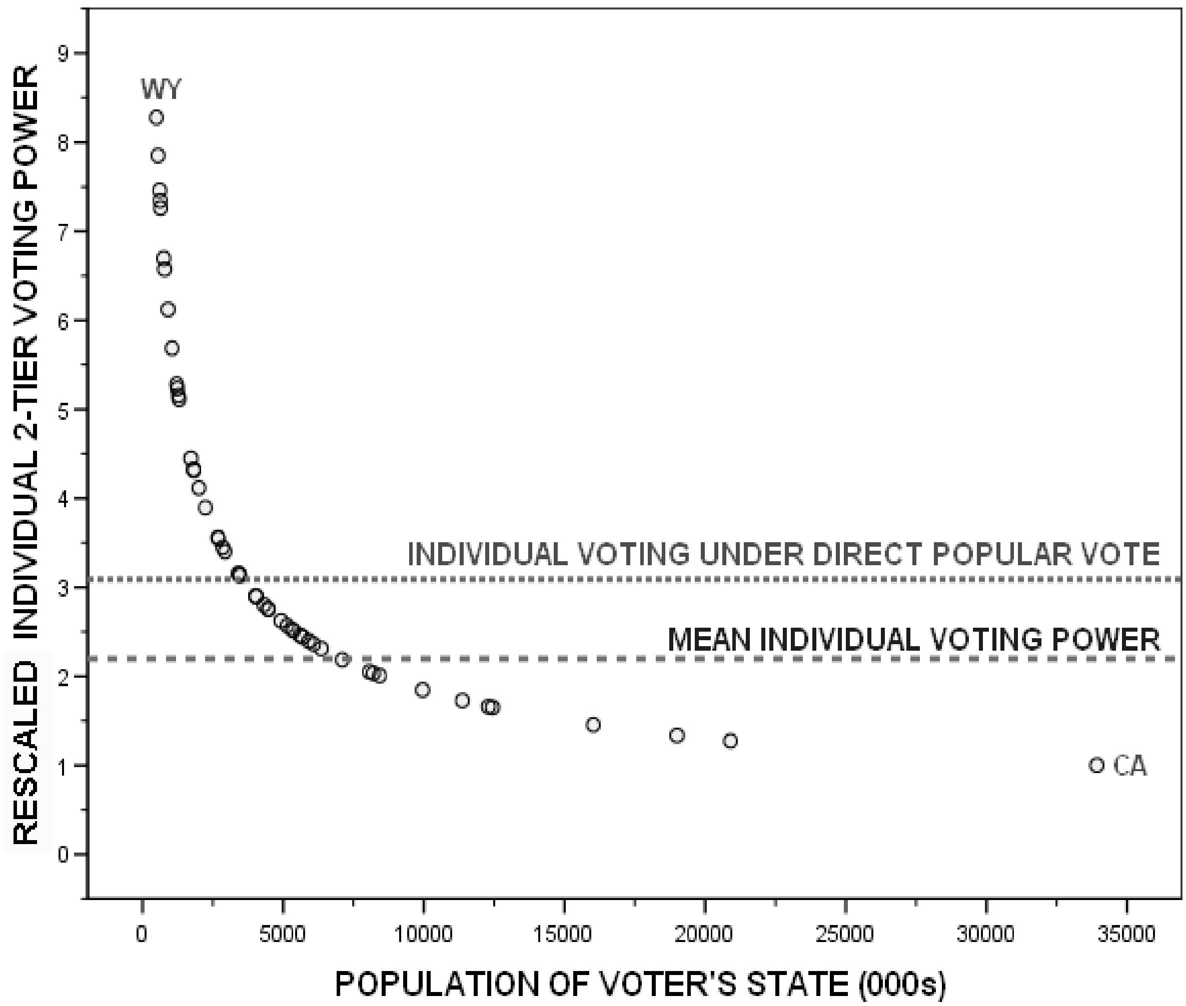


Figure 9 Individual Voting Power by State Population When States Have Equal Electoral Votes

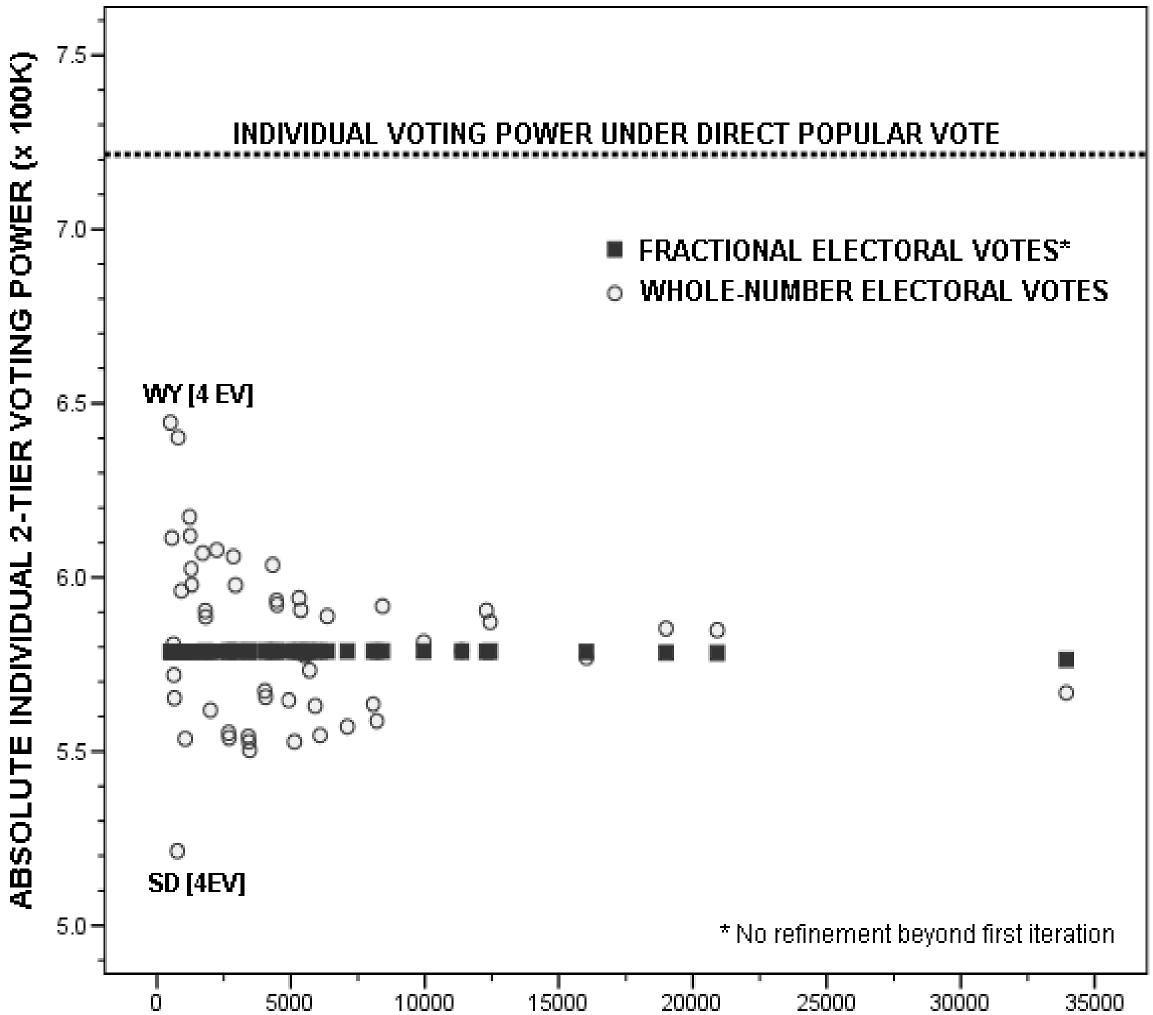


Figure 10 Individual Voting Power by State Population under Penrose Apportionment

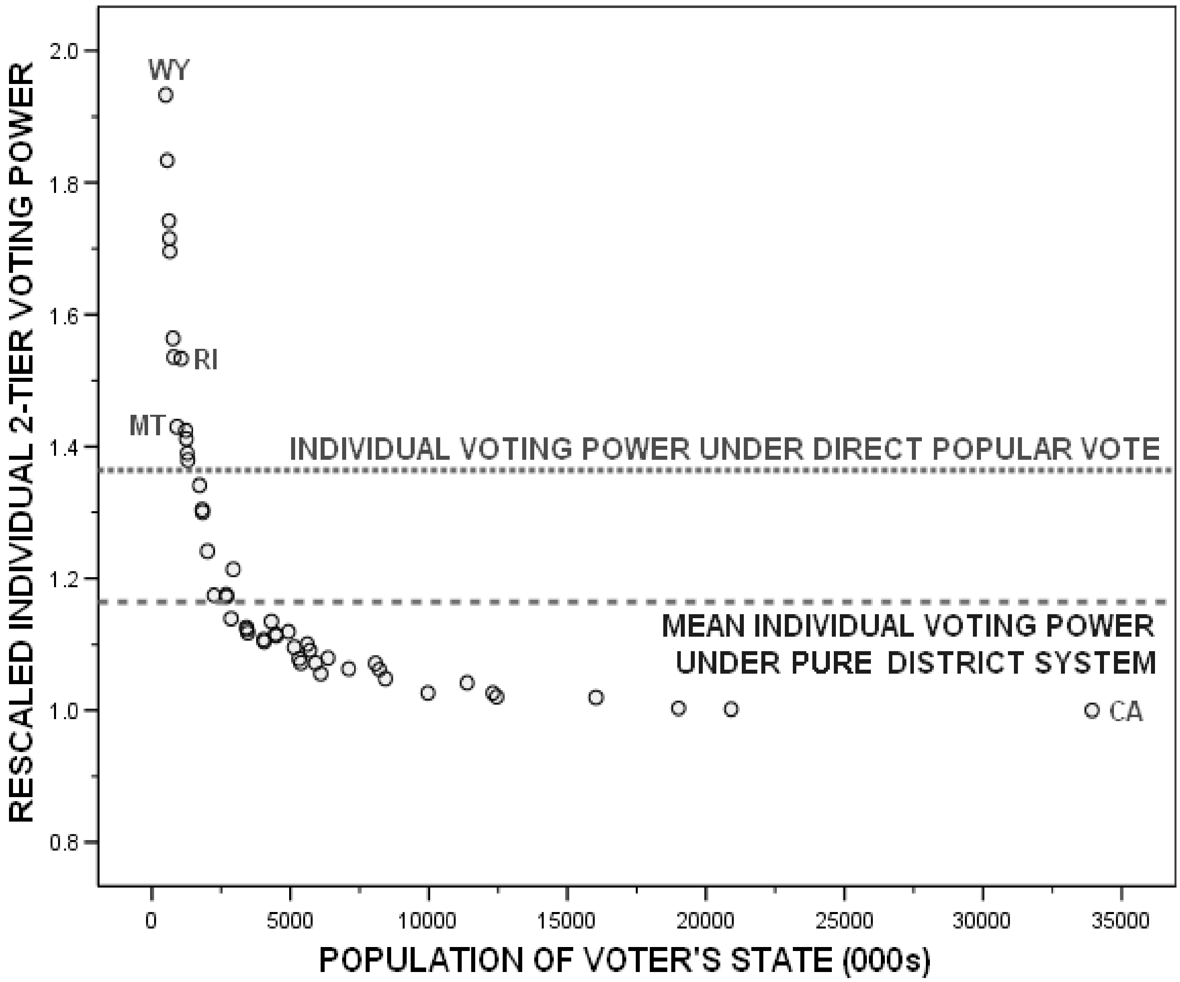


Figure 11 Individual Voting Power by State Population under the Pure District Plan

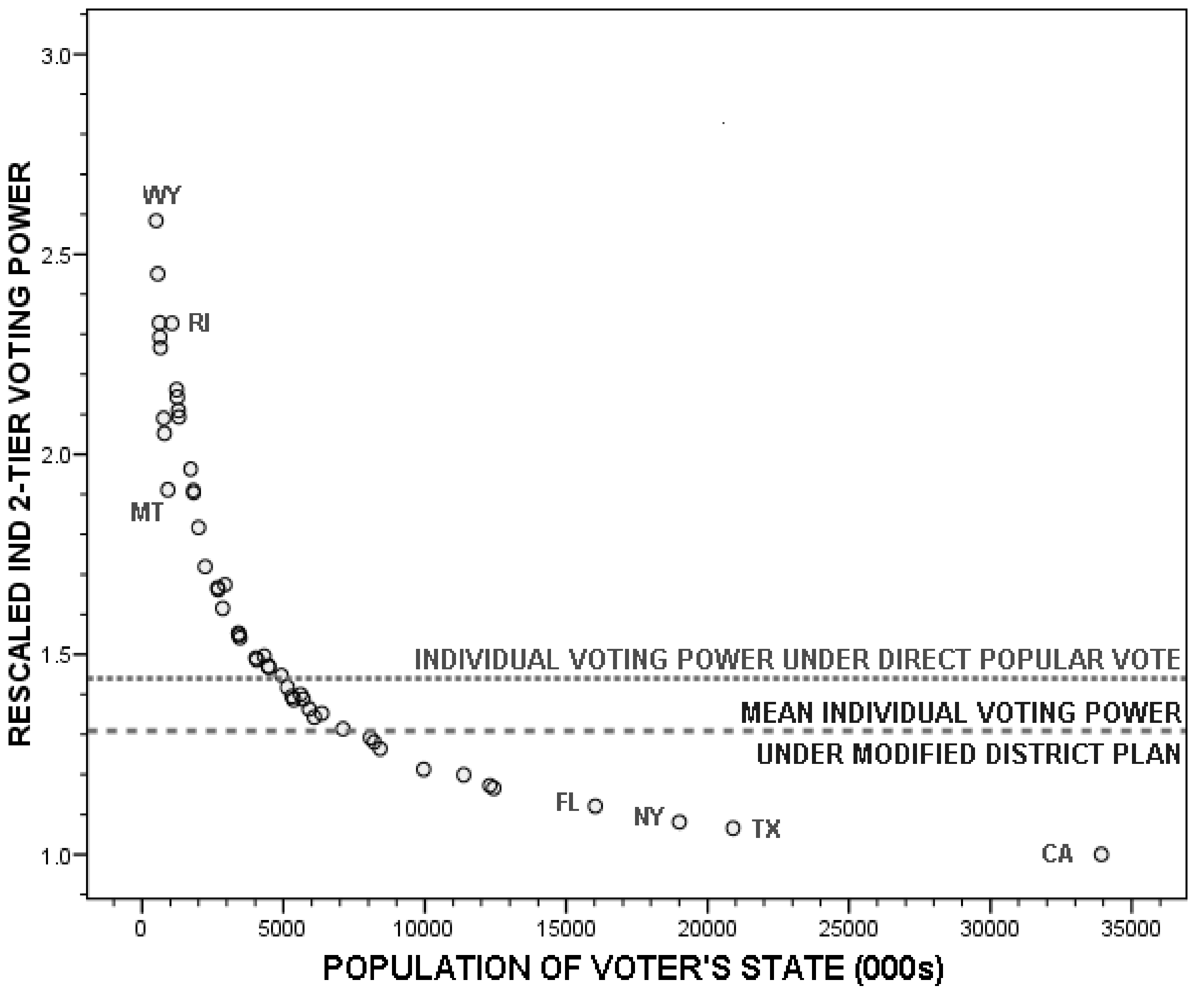


Figure 12 Individual Voting Power by State Population under the Modified District Plan

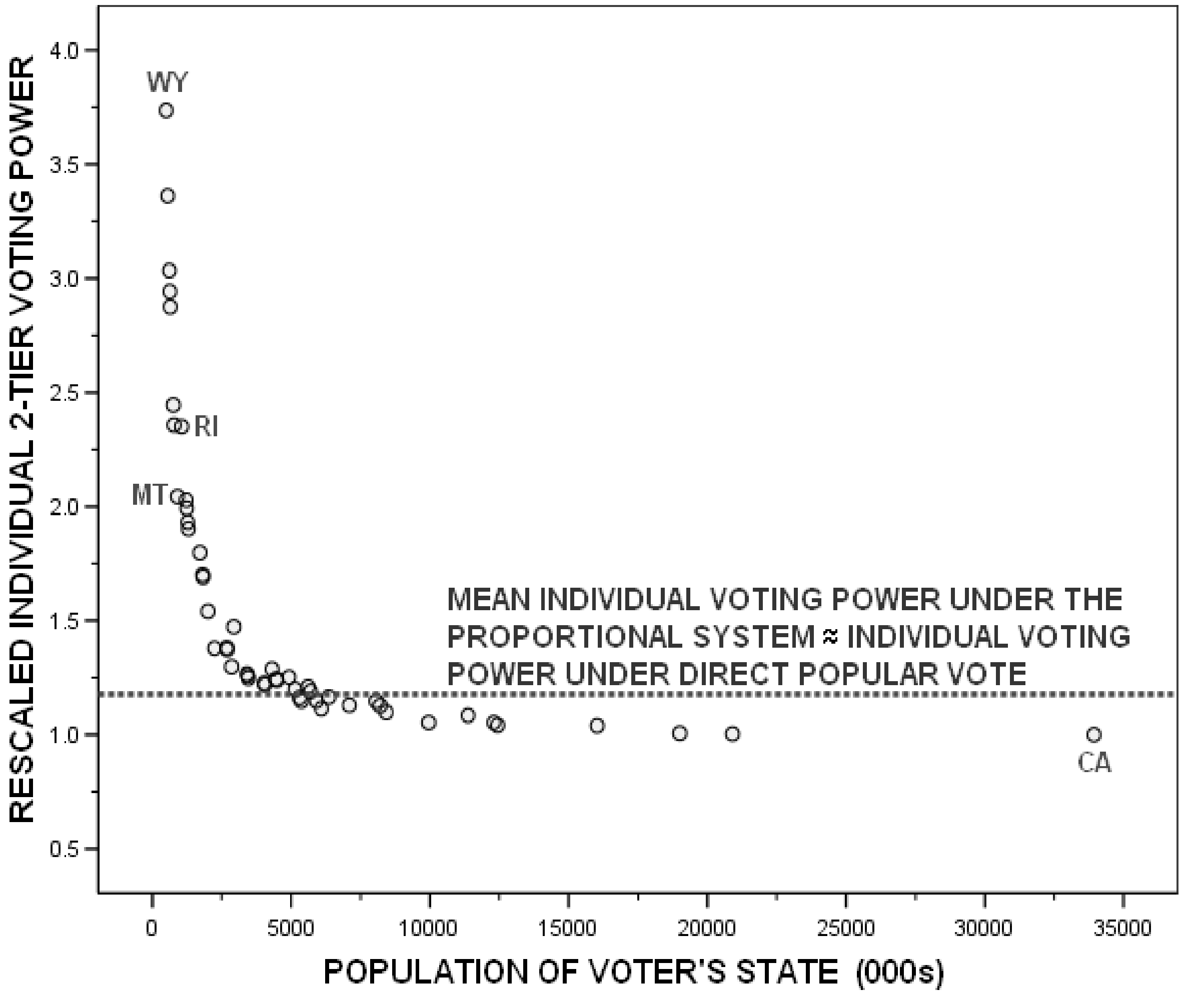


Figure 13 Individual Voting Power by State Population under the Pure Proportional Plan

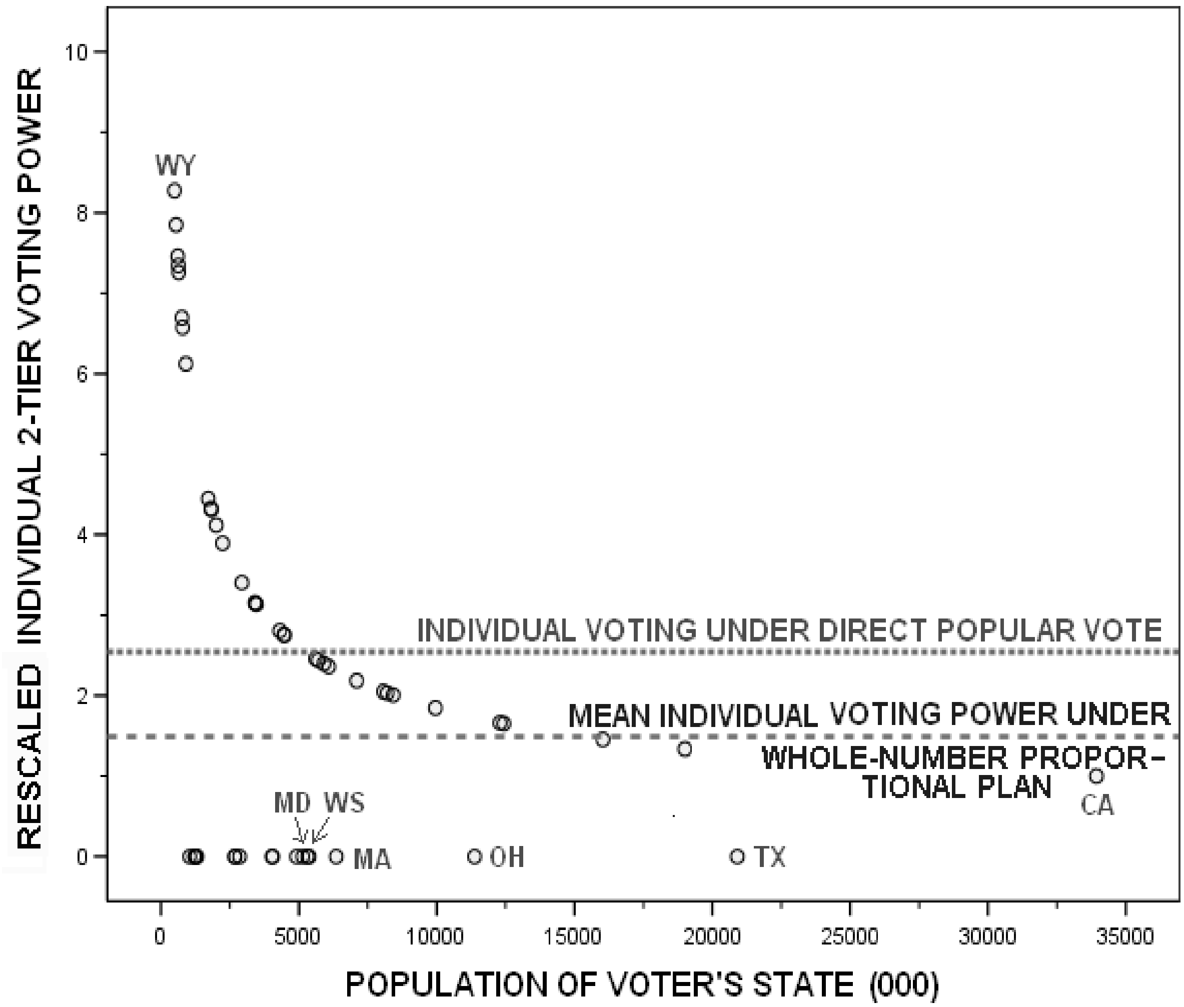


Figure 14 Individual Voting Power by State Population under the Whole-Number Proportional Plan

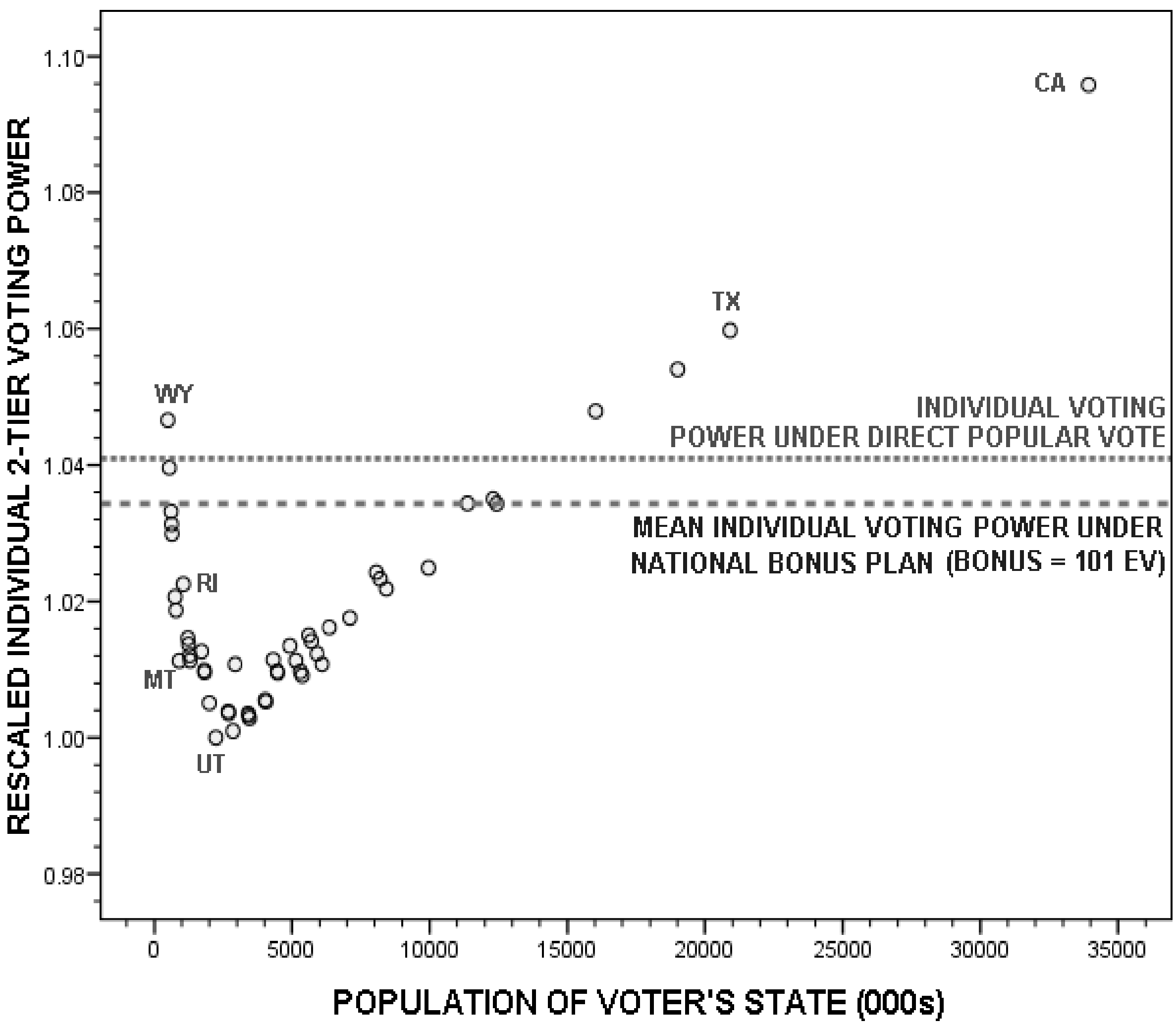


Figure 15A Individual Voting Power by State Population under the National Bonus Plan (Bonus = 101)

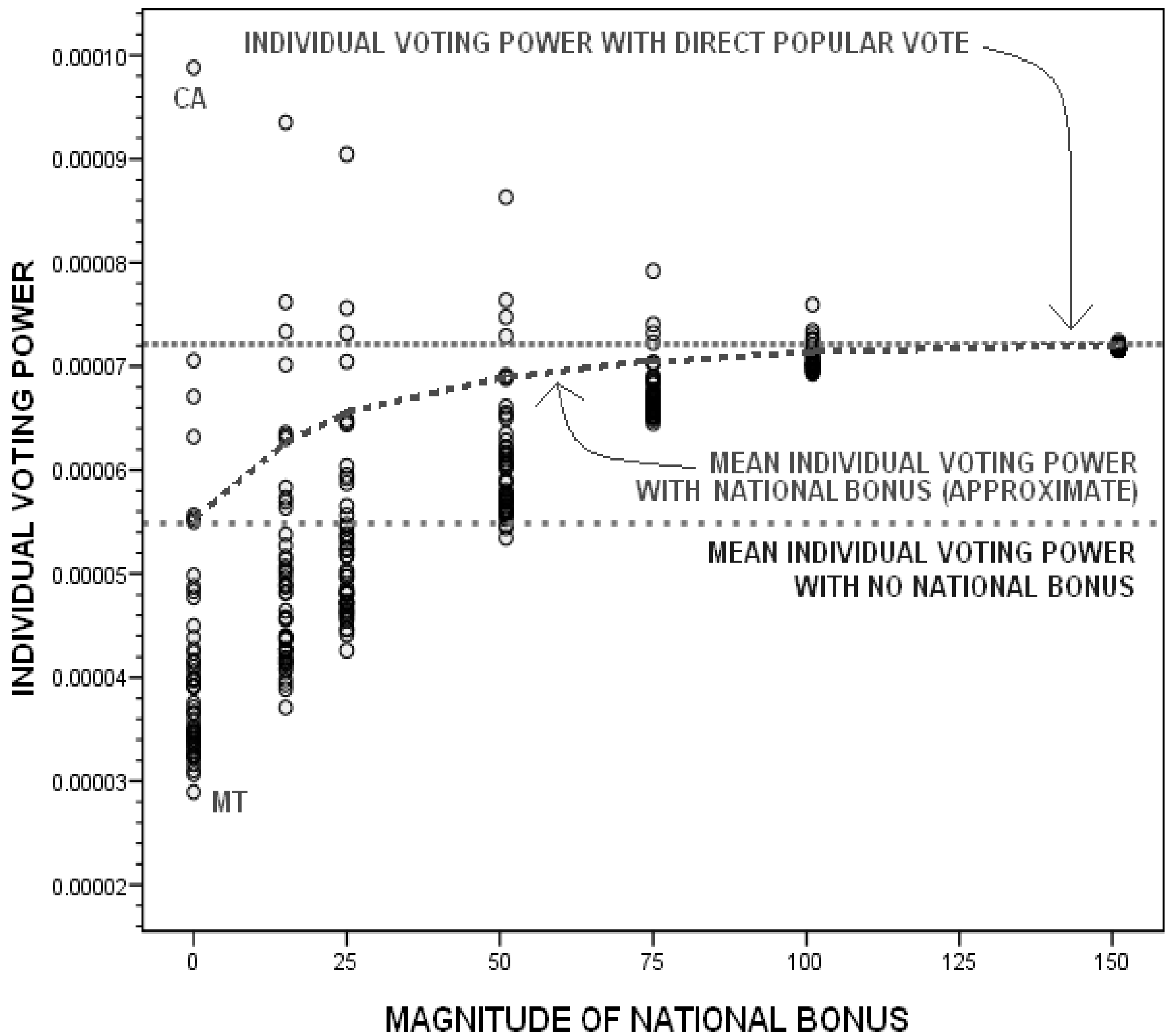


Figure 15B Individual Voting Power by State Population under the National Plan (with Varying Bonus)

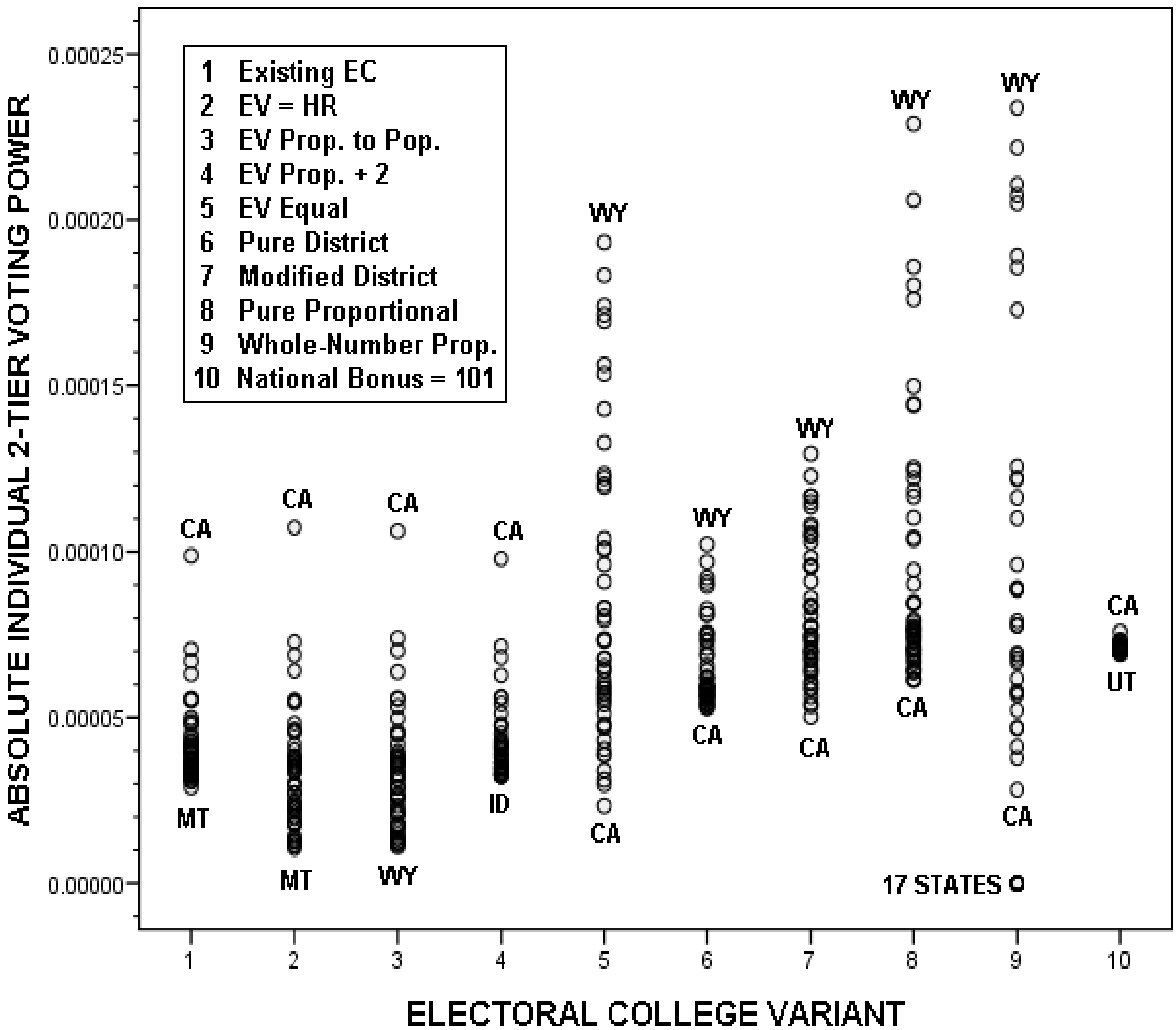


Figure 16 Summary: Individual Voting Power under Electoral College Variants

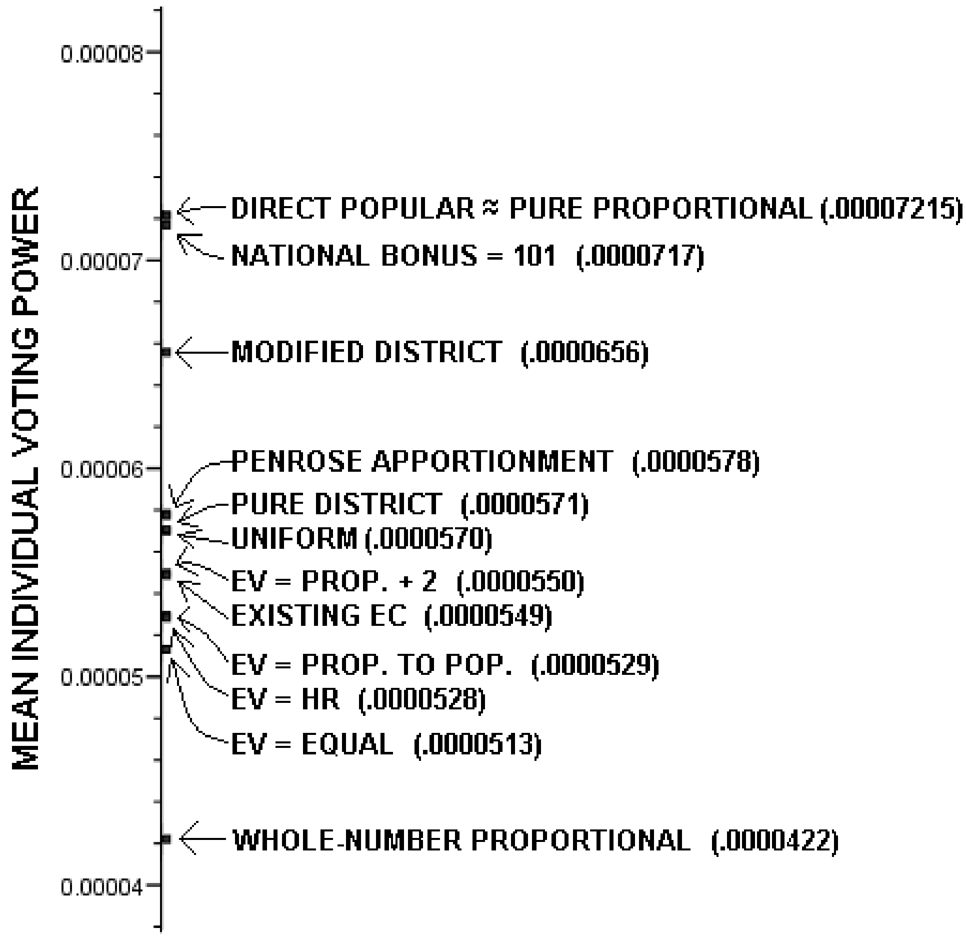


Figure 17A Mean Voting Power under Electoral College Variants

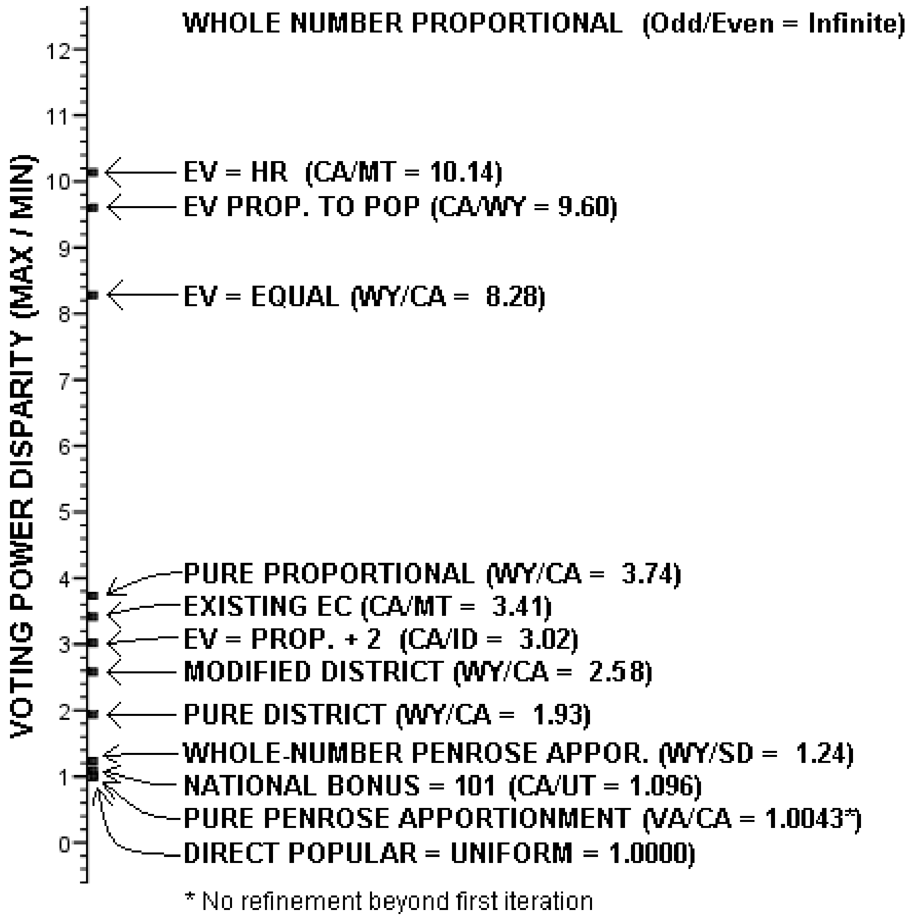


Figure 17B Maximum vs. Minimum Voting Power under Electoral College Variants

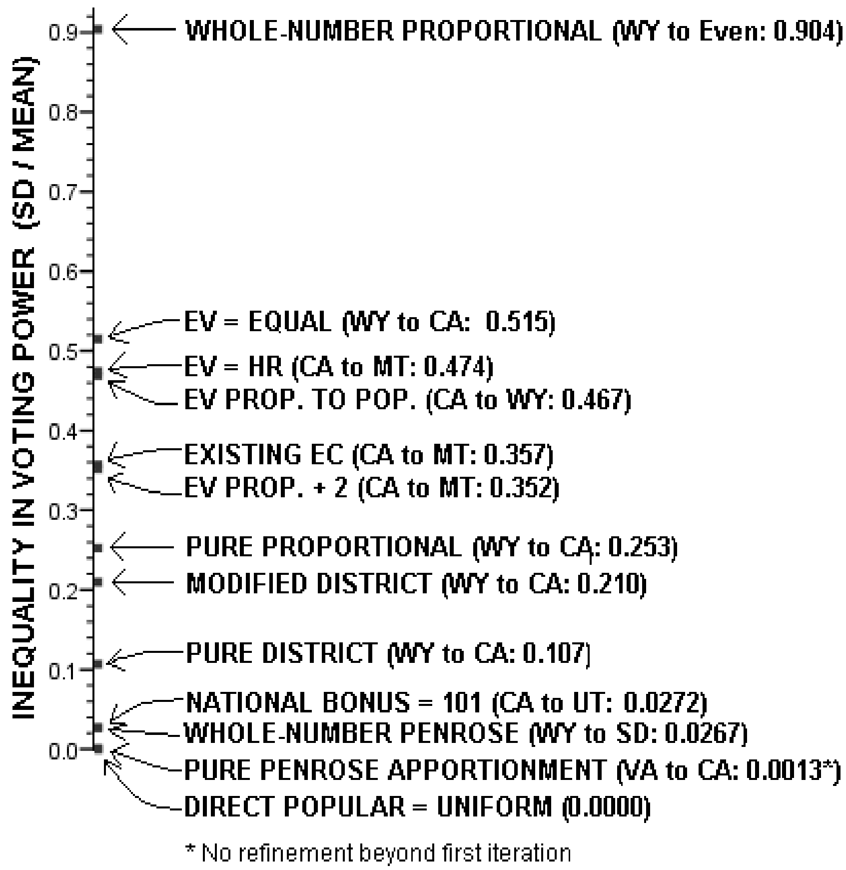


Figure 17C Inequality in Voting Power under Electoral College Variants

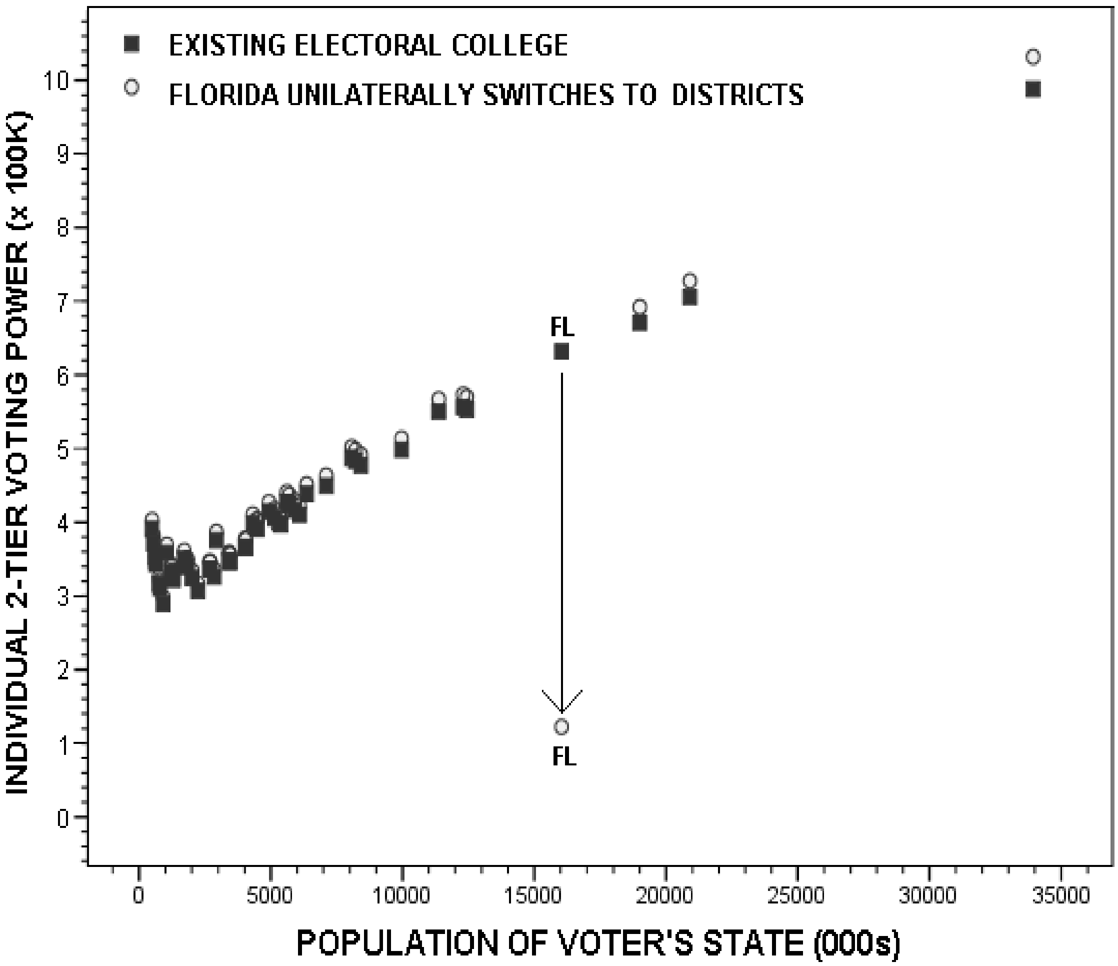


Figure 18A Individual Voting Power: Florida Switches From Winner-Take-All to Pure District Plan

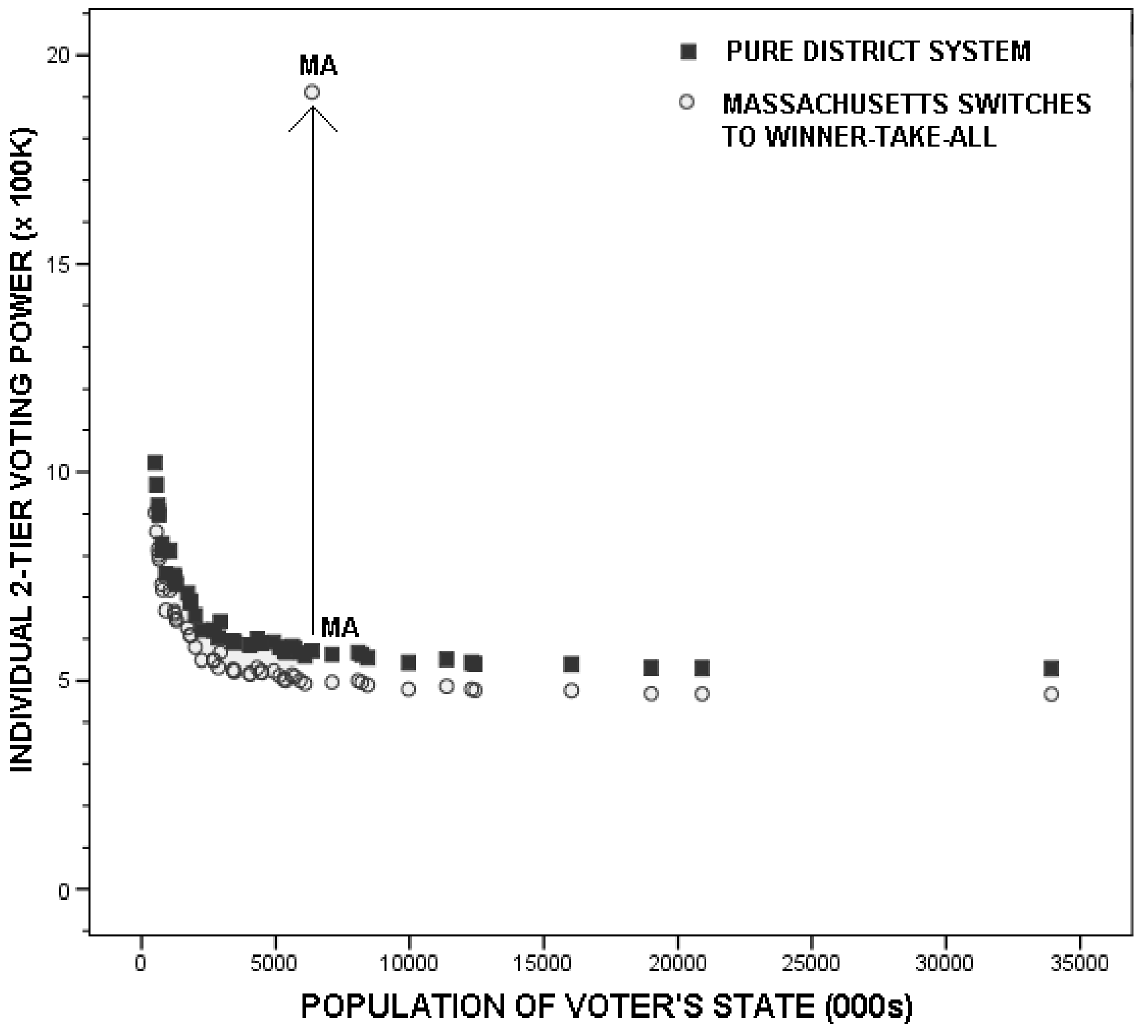


Figure 18B Individual Voting Power: Massachusetts Switches From Pure District Plan to Winner-Take-All