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ABSTRACT

Title of Document:	GAMES BASED ON KNOT MOSAICS
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This thesis will examine the knot mosaic system developed by Samuel Lomonaco and Louis Kauffman and apply the concept to playable games. In the original work, Lomonaco and Kauffman describe a tile set that can be used to represent knots, but the effectiveness of the tile set in terms of building knots and teaching readers about the mosaic system was not examined. The purpose of this thesis will be to define a modified version of the knot mosaic system that allows for users to build the mosaic piece by piece, introduce basic knot transformations, and create games that can be played to teach the players about knot mosaics. The final result will examine the games' abilities to teach players about the subject and measure how enjoyable the games are to play based on player feedback, as well as propose ideas for improvements upon the game rules and mechanics.

GAMES BASED ON KNOT MOSAICS

By

Joseph James O'Malley IV

Thesis submitted to the Faculty of the Graduate School of the
University of Maryland, Baltimore County, in partial fulfillment
of the requirements for the degree of
Master of Science in
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Chapter 1: Introduction to Knot Mosaics

1.1 Defining a Knot Mosaic

The goal of this paper is to introduce and describe playable games that can be used to teach players about knot mosaics and about knot transformations. In order to understand the games and the main mechanics detailed in this paper, a definition of a knot mosaic and what a knot mosaic is made of must be clearly made. Lomonaco and Kauffman (2008) describe a knot mosaic as "a graded set of symbol strings" (p. 2). The symbol strings in the knot mosaics consist of eleven total symbols, which are called mosaic tiles. These eleven symbols are created from five distinct mosaic tiles, which are the result of rotating the five distinct tiles.

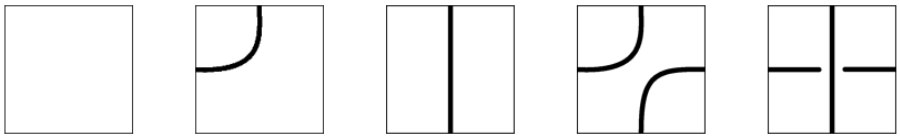

Figure 1.1: The five distinct mosaic tiles

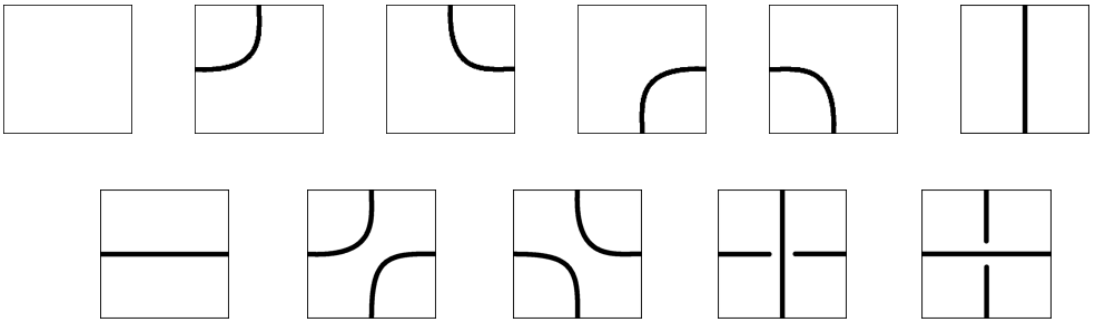

Figure 1.2: The eleven mosaic tiles used for knot mosaics, created by rotating the five distinct tiles above

The mosaic tiles can be connected to one another by what are called connection points. Lomonaco and Kauffman (2008) define a connection point of a mosaic tile as "the midpoint of a tile edge which is also the endpoint of a curve drawn on the tile" (p. 5). In Figure 1.3, all possible numbers of connection points for any given mosaic tile are shown, which means that any given tile must have 0, 2, or 4 connection points. The connection points allow the mosaic tiles to connect to one another in order to create a complete knot mosaic.

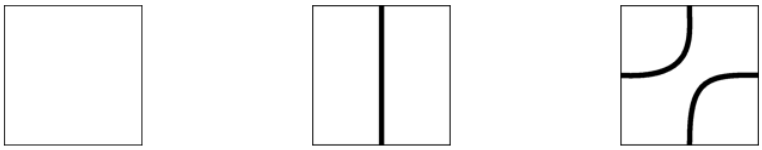

Figure 1.3: Three different mosaic tiles with varying numbers of connection points

The original definition for the knot mosaic system was described by Lomonaco and Kauffman (2008) as a knot n-mosaic, which is a mosaic "in which all tiles are suitably connected" (p. 3). In order for a mosaic tile to be suitably connected, it must be connected to other mosaic tiles at all of its connection points. However, if all tiles in a mosaic are suitably connected, then the mosaic is closed with no room to add more tiles on the edges of the mosaic. Due to this restriction, a modified definition of the knot n-mosaic will be used to describe mosaics that are not closed and allow adding new mosaic tiles to the edges of the mosaics. The new type of mosaic that will be used is called an open knot mosaic, which is a combination of mosaic tiles on an n x n grid that are connected via their connection points where the interior tiles are suitably connected but the exterior edges need not be suitably connected.

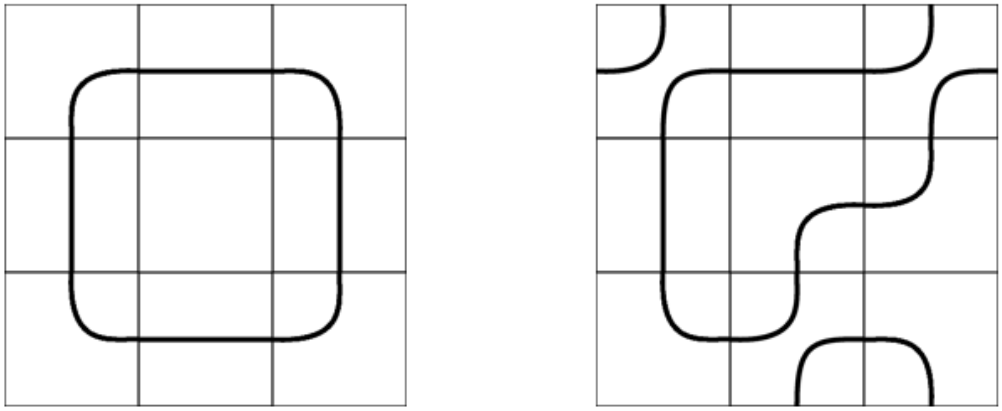

Figure 1.4: An example of a knot n-mosaic and an open knot mosaic

Figure 1.4 gives examples of both a basic knot n-mosaic and an open knot mosaic. The 3 x 3 knot n-mosaic, shown on the left in Figure 1.4, consists of tiles that are all suitably connected, thus making the mosaic closed and unable to be built upon. On the contrary, the 3 x 3 open knot mosaic, shown on the right in Figure 1.4, consists of tiles that are suitably connected and tiles that are not suitably connected. Due to the inclusion of tiles that are not suitably connected, it is possible to add more tiles to the open knot mosaic by using the connection points on the tiles that are not suitably connected. This characteristic allows game scenarios to be made by allowing the players to build open knot mosaics piece by piece.

1.2 Special Tiles and Moves

Now that the basics of the knot mosaic system have been covered, it is possible to implement advanced techniques and modified tile sets that can be used in knot mosaic games. Such techniques and tile sets will allow for more varied play conditions and options for the players outside of only placing tiles on the open knot mosaic. The games that will be described will utilize a general mechanic found in basic knot

theory, an extra tile set described by Lomonaco and Kauffman, and two Reidemeister Moves that have been translated into the mosaic tiling system that can be easily implemented as basic game mechanics.

The first feature to be implemented is the concept of smoothing. Denis A. Fedoseev and Vassily Manturov (2017) describe the smoothing of a one-dimensional crossing as "cutting out a crossing with its small [neighborhood] and [regluing] two pairs of half-edges in one of the possible two ways" (p. 5). Due to the nature of the mosaic tile system, the concept of smoothing is simple to implement and allows for a basic transformation that the players are able to utilize within the game scenarios, as well as extending the definition to allow for the half-edges to transform into a crossing. The smoothing transformation can be seen in Figure 1.5, which shows the relationship between tiles that represent a crossing and a tile that represents the half-edge.

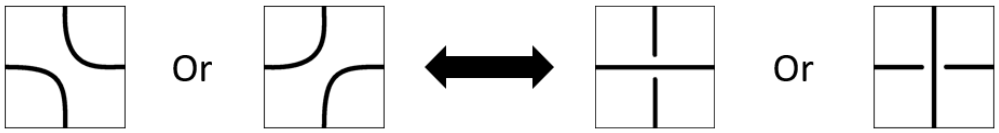

Figure 1.5: The smoothing technique for knot mosaics

The second feature to be implemented is the nondeterministic tile set described by Lomonaco and Kauffman (2008, p. 9). The tiles included in the nondeterministic tile set have the special property of representing two or more different mosaic tiles at the same time. For any given nondeterministic tile, the dashed lines that appear on the tile represent a piece of rope that either occupies the given spot on the tile or an area with no piece of rope present. This rule follows for any nondeterministic tile in which the

dashed lines do not overlap, which can be seen in the examples given in Figure 1.7, but tiles that have overlapping dashed lines represent a slightly different set of tiles. If a given nondeterministic tile contains overlapping dashed lines, then the mosaic tiles that are represented by the tile must contain one and only one of the dashed lines, which can be seen in Figure 1.8. Because of this special case, any nondeterministic tile piece with overlapping dashed lines cannot represent a blank mosaic tile.

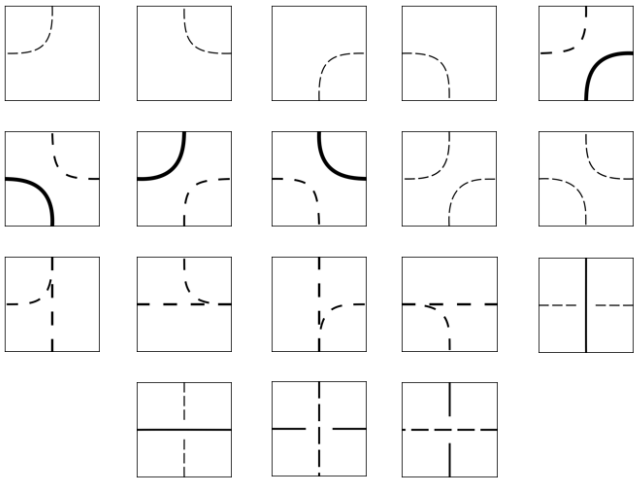

Figure 1.6: The nondeterministic tile set

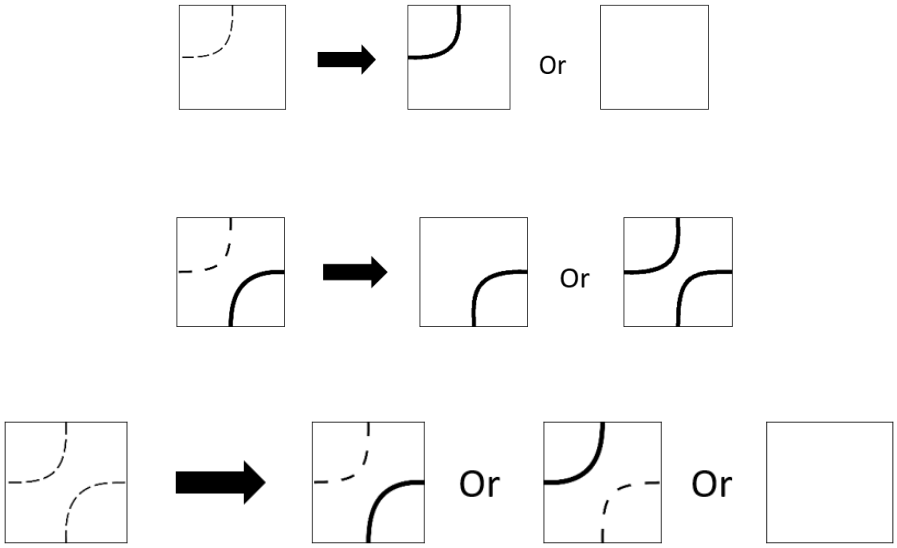

Figure 1.7: Some examples of nondeterministic tiles and what they can represent

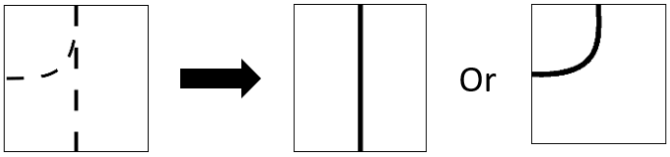

Figure 1.8: An example of overlapping nondeterministic lines, where a blank tile is not a possible result

The next features that will be implemented into the games are the two types of Reidemeister moves modified for use in the mosaic tile system. The Reidemeister moves for classical knot theory, created by Kurt Reidemeister, consist of three ways to move segments of rope such that a related diagram can be created. Lomonaco and Kauffman translated the three Reidemeister moves to the knot mosaic system (2008, p. 12-13), but for the sake of simplicity, the games described will implement two of the three Reidemeister moves. Like the smoothing technique, these Reidemeister moves will allow players to make changes to the board after pieces are already placed, but the Reidemeister moves allow for longer term strategies since they are a bit more complex to set up.

The first move to be implemented is the Reidemeister 1 move, which will be derived from Lomonaco and Kauffman's mosaic Reidemeister 1 move (2008, p. 12) and can be classically implemented (Wolfram MathWorld). The mosaic Reidemeister 1 move is a technique that either creates or removes a crossing over a single piece of rope. A depiction of the mosaic Reidemeister 1 move can be seen in Figure 1.9, which shows how a mosaic changes when the move is applied. While the mosaic Reidemeister 1 move performs a transformation similar to that of smoothing, it is important to note that the Reidemeister 1 move depends on the tiles around the transformed piece to show that only one piece of rope is being modified.

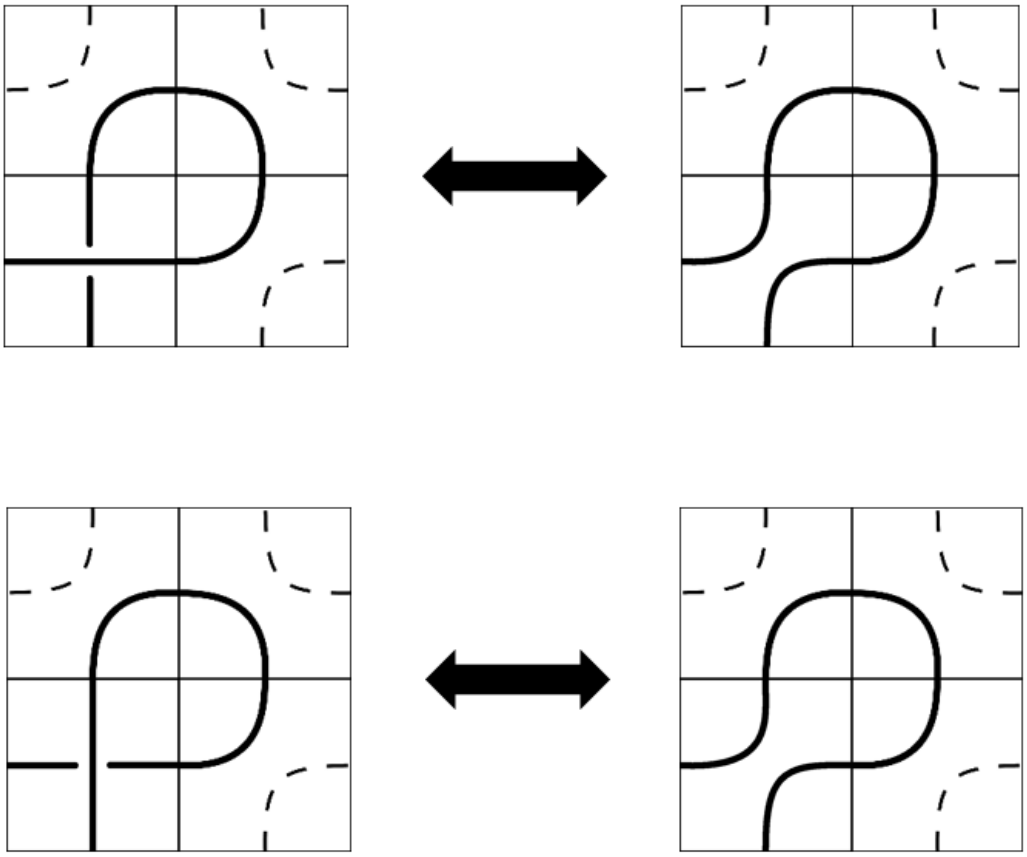

Figure 1.9: The mosaic Reidemeister 1 move

The second move and final special technique to be implemented is the Reidemeister 2 move, which consists of moving one segment of rope completely over a segment of a different rope as seen in the classical implementation of the move (Wolfram MathWorld). Compared to the Reidemeister 1 move previously mentioned, the Reidemeister 2 move utilizes two crossings or two half-edges in order to represent two different pieces of rope. Figure 1.10 shows the Reidemeister 2 move and all possible variations of the move, as detailed by Lomonaco and Kauffman (2010, p.12).

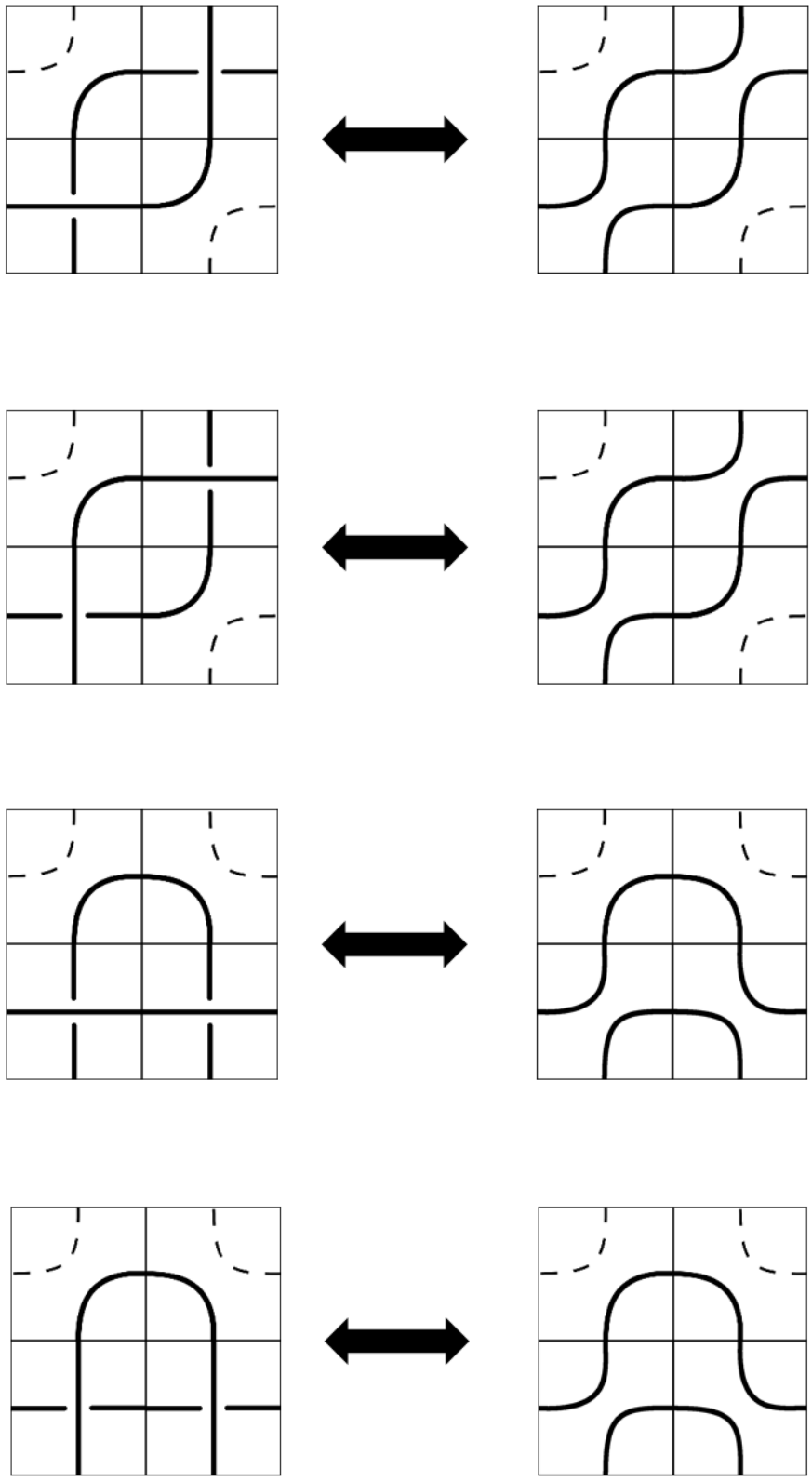

Figure 1.10: The mosaic Reidemeister 2 move

Chapter 2: Knot Mosaic Game Scenarios

2.1 Introducing Knot Mosaic Games and General Mechanics

With the general structure of open knot mosaics and some special moves described, it is possible to create multiple games that teach certain aspects about the knot mosaic system. While different games may have different rules or win conditions, the games will follow some basic rules that represent the base gameplay.

At the start of a game, no pieces will be on the board. The turn order for the game will be decided by rock-paper-scissors, with the winner going first and the players alternating turns for the duration of the game. On the first turn, during which there are no pieces on the board, any piece may be placed in order to start. After the first turn, any piece that is added to the board must be connected by an open connection point on the board. The board has dimensions $\infty \times \infty$, which allows the players to build the mosaic in any direction if there is room to add new pieces. The players are able to do one of two kinds of moves on each of their turns, which will be classified as placing a piece or transforming a piece. Placing a piece follows the rules of normally putting a piece on the board via the available connection points, whereas transforming a piece will allow the player to use the smoothing technique, use a Reidemeister move, or define a nondeterministic piece in order to modify a piece that is already on the board. In order to avoid loops of transformations, if a board piece is transformed in the same way for four consecutive turns, the game ends in a draw. After some win condition is met, which is determined by the game scenario, the game ends and the player that fulfilled the win condition is determined the winner.

2.2 Create the Picture Game Scenario

The first type of game is one that tasks the players with creating a knot mosaic that they are given at the start of the game. The players will know what mosaic that they must build, but will not know what their opponent must build, allowing players to study how their opponent is building the board and attempt to prevent them from building their mosaic. The game ends once a section of the board represents the knot mosaic that one of the players is given, with that player being declared the winner.

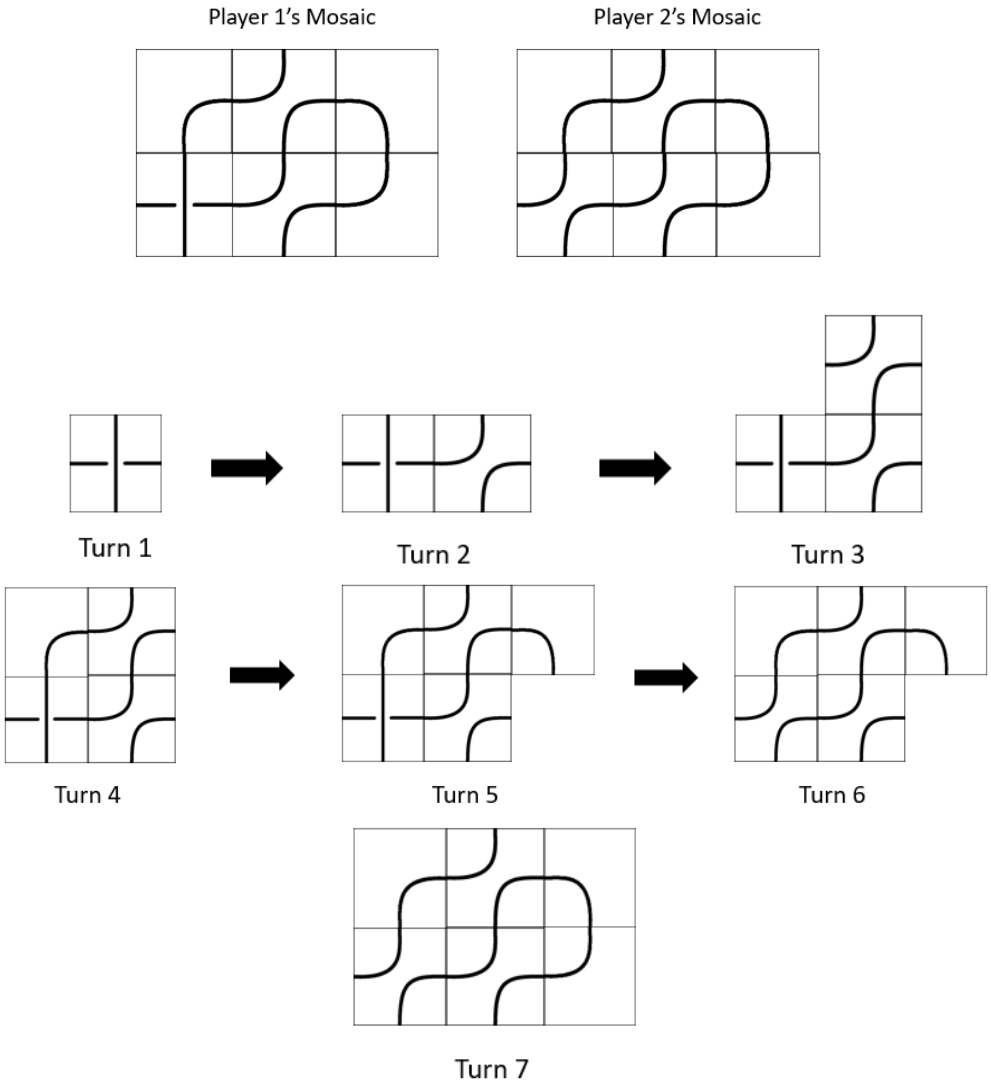

Figure 2.1: A sample game played with the Create the Picture rule set

Figure 2.1 showcases a sample run-through of a game that tasks the players to make a knot mosaic that they are given. In this example, as well as all other game examples, player 1 makes the first move. From turn 1 to turn 5, the players decide to build in any direction of their choosing while also making progress towards their given knot mosaics. By turn 5, player 1 is one piece away from achieving victory, but on turn 6, player 2 sees that it is possible to use the smoothing technique in order to change the board in their favor. Due to not knowing what mosaic player 2 needs to build, player 1 decides to place the final corner piece and ends up completing player 2's mosaic by mistake. With player 2's mosaic built, the game is over and player 2 is declared the winner.

The game presented in Figure 2.1 shows off two major game mechanics, which are the smoothing technique and the importance of the opponent's mosaic being concealed. The smoothing technique allowed player 2 to make a change to the board without needing to set up for a Reidemeister move, which changed the game from favoring player 1 to favoring player 2 at the moment. While the smoothing could have been done earlier in the game, its effects are the most pronounced the closer a player is to winning the game, keeping the players alert at all points of the game. Concealing the players' mosaics is shown to have an important effect at the end of the game, when player 1 accidentally completes player 2's mosaic. By concealing the mosaics, the players are tasked with putting more thought into their moves by studying how their opponent is playing as well as coming up with their own plan of building their mosaic. Player 2 using the smoothing technique on turn 6 was a sign that the move was important, but player 1 may not have thought much about it and

decided to build his general mosaic structure without regards to player 2's mosaic. Overall, the Create the Picture game allows the players to easily pick up the game and learn the basic game mechanics while having the solution mosaics guide the players towards a desired outcome.

2.3 The Point Game

The second type of game is one that is similar to the Create the Picture game, but changes the win condition in order to emphasize the usage of special moves. In this game scenario, the players are given a mosaic that they must create, but the winner is decided by how many points the players accumulate over the course of the game, not by creating the player's target mosaic. The points in this game are earned by the usage of the smoothing technique, the Reidemeister moves, and by completing the target mosaic that the player is given. The point values for each of the sources will be as follows: the smoothing technique is worth 1 point, both Reidemeister moves are worth 2 points, and a player completing their target mosaic is worth 3 points. In order to prevent players from continuously obtaining points from a single source the entire game, players will not be able to receive points from a tile, or set of tiles, that they have already use for a given transformation. For example, if player 1 uses the smoothing technique on a crossing piece, that player will not be able to receive any more points if they use the smoothing technique on the same piece, but can use either Reidemeister move with that crossing piece to receive points once for each move. Once a player creates their target mosaic, the game is over and the points are tallied up, with the player having the most points being declared the winner. If the players have the same point total at the end of the game, the game is declared a tie.

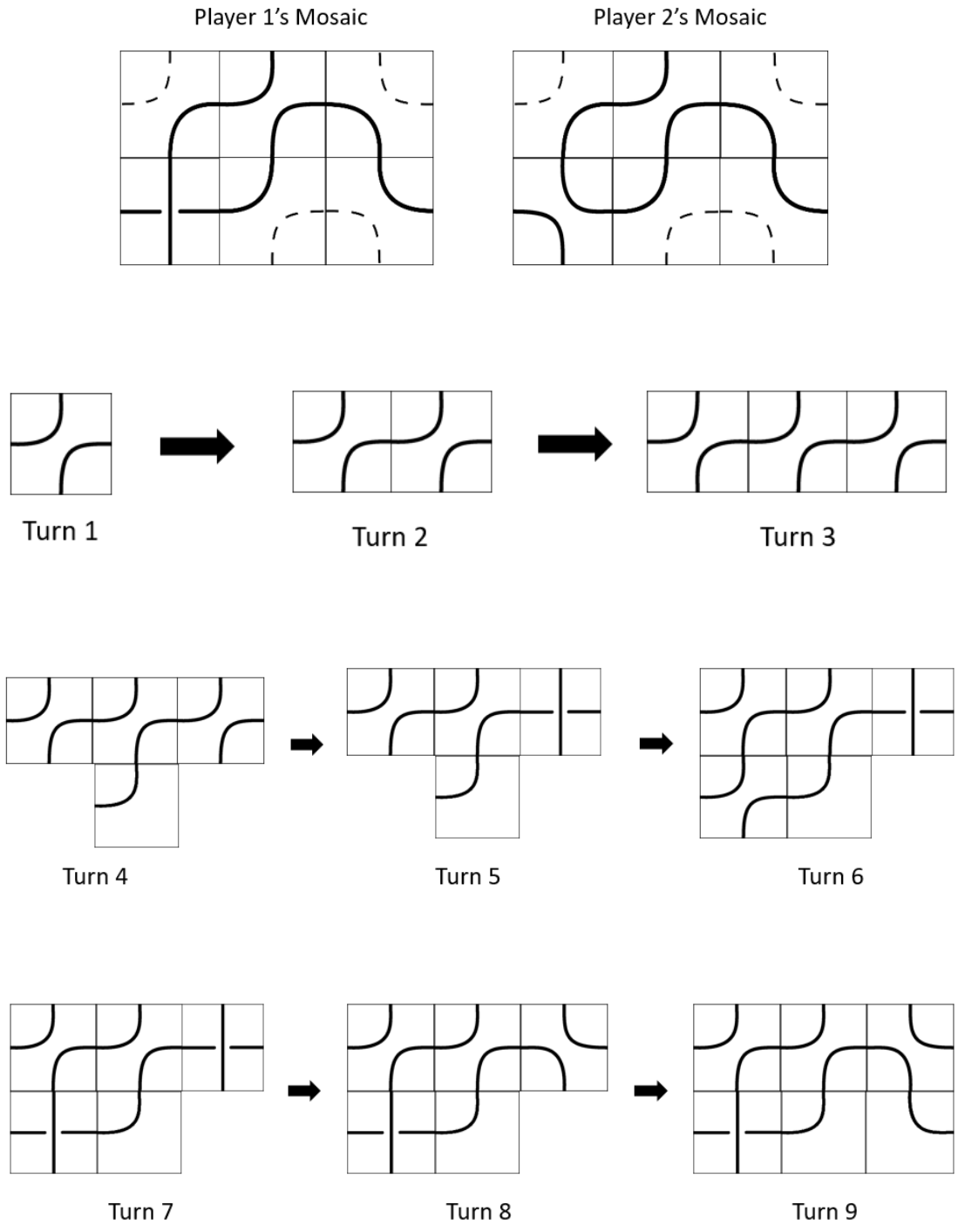

Figure 2.2: A sample game played with the Point Game rule set

Figure 2.2 shows an example run-through of the Point game. Both players spend the first few turns placing tiles that can fit with their target mosaics, but player 1 takes advantage of the smoothing technique on turn 5 and turn 7 to earn points and get closer to their target mosaic. While it was possible to make a Reidemeister 2 move on turn 7, it would take longer for the player to reach their target mosaic if the move was made, which may give player 2 time to earn points and take the lead. Turn 8 shows player 2 using a smoothing to earn points and get close to their target mosaic, but player 1 is able to finish their mosaic the next turn due to player 2's move. At the end of the game, player 1 earned five points in total, two from the smoothing of two different tiles and three from completing their mosaic, and player 2 earned one point in total from a single smoothing. Player 1 is declared the winner of this game with a final score of 5 to 1 in favor of player 1.

The game in Figure 2.2 shows off three major mechanics, which are the Reidemeister 2 move, the importance of limiting the points gained from repeat moves on tiles in the Point game scenario, and the nondeterministic tile set. Although it did not see use in the given example, the Reidemeister 2 move was set up for use for a larger point gain compared to the smoothing technique. Despite this, player 1 opted to not use it since it would allow them to get their target mosaic faster, potentially limiting player 2's point gain in the process. Limiting the points gained from repeat moves on tiles, while not a pronounced feature, does help to guide the players towards building the mosaic instead of constantly changing tiles that are currently on the board. If this rule were not in place, then the players could spend far more time flipping tiles for unlimited points instead of focusing on the main goal of the game.

Lastly, the nondeterministic tiles are introduced in this game scenario and show just how they can open up different strategies, especially when used in target mosaics. By including nondeterministic tiles in target mosaics, the possible combinations of tiles that can be used to create a correct solution increases, allowing the players to have more freedom in terms of what tiles they can add or manipulate while still making progress towards their target mosaics. Even though it is possible for the players to use the nondeterministic tiles in normal play, their effects are most pronounced when implemented in the target mosaics to allow for more varied solutions. Overall, the Point game introduces a simple yet unique modification to the basic Create the Picture game that emphasizes the usage of special moves, the setting up of these special moves, and better strategy and planning overall in order to gain more points over the course of the game.

2.4 The Colored Rope Game

The next type of game strays away from the previous two game scenarios by introducing a different color of rope. In this game scenario, there will be two different ropes: a black rope and a gray rope. Player 1 will only be able to place tiles that include a piece of black rope, whereas player 2 is given the same restriction but for tiles that include a piece of gray rope. The first turn of the match must have a player place a tile that contains both a black rope and a gray rope. The win conditions for this game are the same as the Create the Picture game, which is to say that the players must create a target mosaic that they are given.

The colored rope tiles function differently compared to the single color tiles that have been used up to this point, so it is important to know how exactly they can be

used and what differences exist between the two tile sets. As previously mentioned, the colored tiles consist of tiles that can contain both black ropes and gray ropes, with both of these representing different pieces of rope. The colored rope tiles consist of pieces that are either fully one color of rope or a combination of both the black rope and the gray rope, with the tile layouts mimicking those found in the base mosaic tile set. When placing the colored rope tiles onto the board, the tiles must be connected via the connection points of the same color. The tiles will also make use of the smoothing technique, the Reidemeister moves, and the nondeterministic tile notation, but these transformations must respect the connection points between ropes of the same color in order to be used. Lastly, in order to promote player interaction, two rules will be put in place for games that use the colored rope tile set. The first rule is that any tiles that represent a crossing or a half-edge must use both rope colors. The second rule is that the tiles do not need to have the nondeterministic notation in order for a player to make use of the nondeterministic tile set. Both of these rules are introduced in order to give the players more options every turn without being completely denied options from their opponents. Figure 2.3 shows examples of the colored rope tiles and how they are affected by these two specific rules.

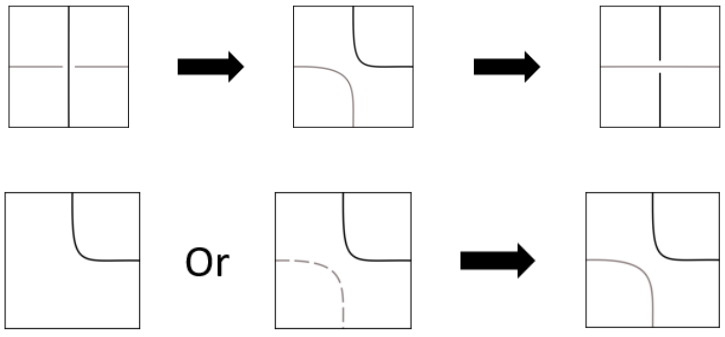

Figure 2.3: The two special cases that are introduced into the colored rope games to allow for greater player interaction.

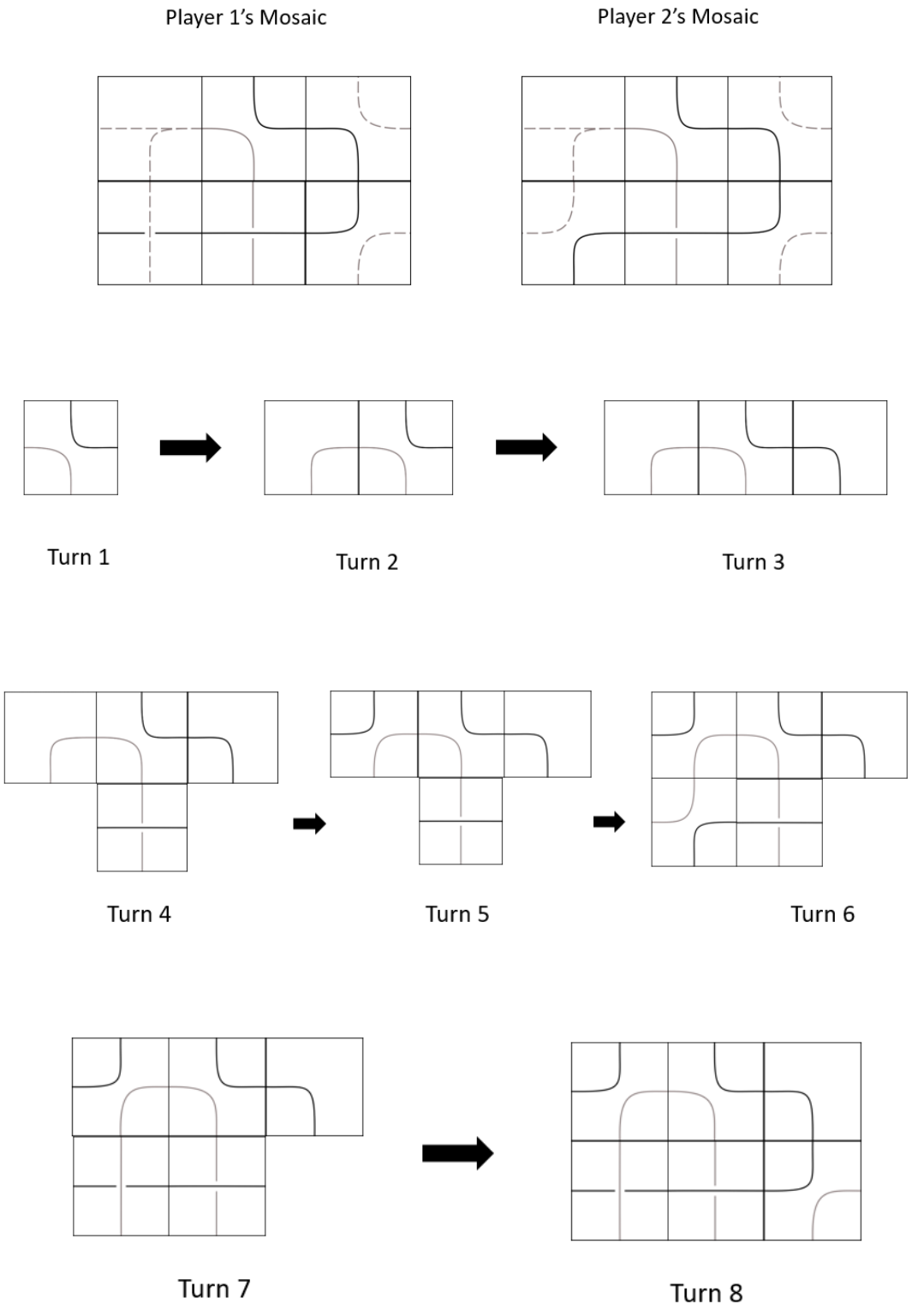

Figure 2.4: A sample game played with the Colored Rope rule set

Figure 2.4 shows an example run-through of the Colored Rope game. The game starts normally, but turn 5 has player 1 making use of the implied nondeterministic tiles to edit a piece on the board. Turn 7 has player 1 using the smoothing technique in order to fulfill their target mosaic and turn 8 has player 2 accidentally giving player 1 their finished target mosaic. With this move by player 2, player 1 is declared the winner.

The example shown in Figure 2.4 shows off the smoothing technique when in a multi-colored rope game, the importance of the implied nondeterministic tiles in the colored rope games, and the usage of the overlapping nondeterministic tiles given to the players in their target mosaics. In order to make use of the smoothing transformation, the new piece must conform to the colored connection points given by the surrounding pieces, otherwise the move cannot be made. The implied nondeterministic pieces and the overlapping nondeterministic piece go hand in hand with one another in this given example. The overlapping nondeterministic piece is utilized in the top left of the game board and ends up as a half-edge at the end of the game. However, due to how the half-edge pieces are defined within colored rope games, the final setup of the piece is valid for a completed target mosaic. Due to these mechanics, a lot more freedom is given to the players to experiment and try new strategies in games featuring different colored ropes.

2.5 The Closed Knot Game

The final game scenario that will be covered is a variation of the colored rope game. Instead of being given a target mosaic to build, the players are instead tasked

with building a closed mosaic of their own rope color. In order to make the game more challenging, the closed knots that the players create must be at least three tiles long vertically, horizontally, or both. The main goal of this game scenario is not to teach the players about the mechanics of knot mosaics, but instead to have the players apply what they have learned in the most freeform and competitive way possible.

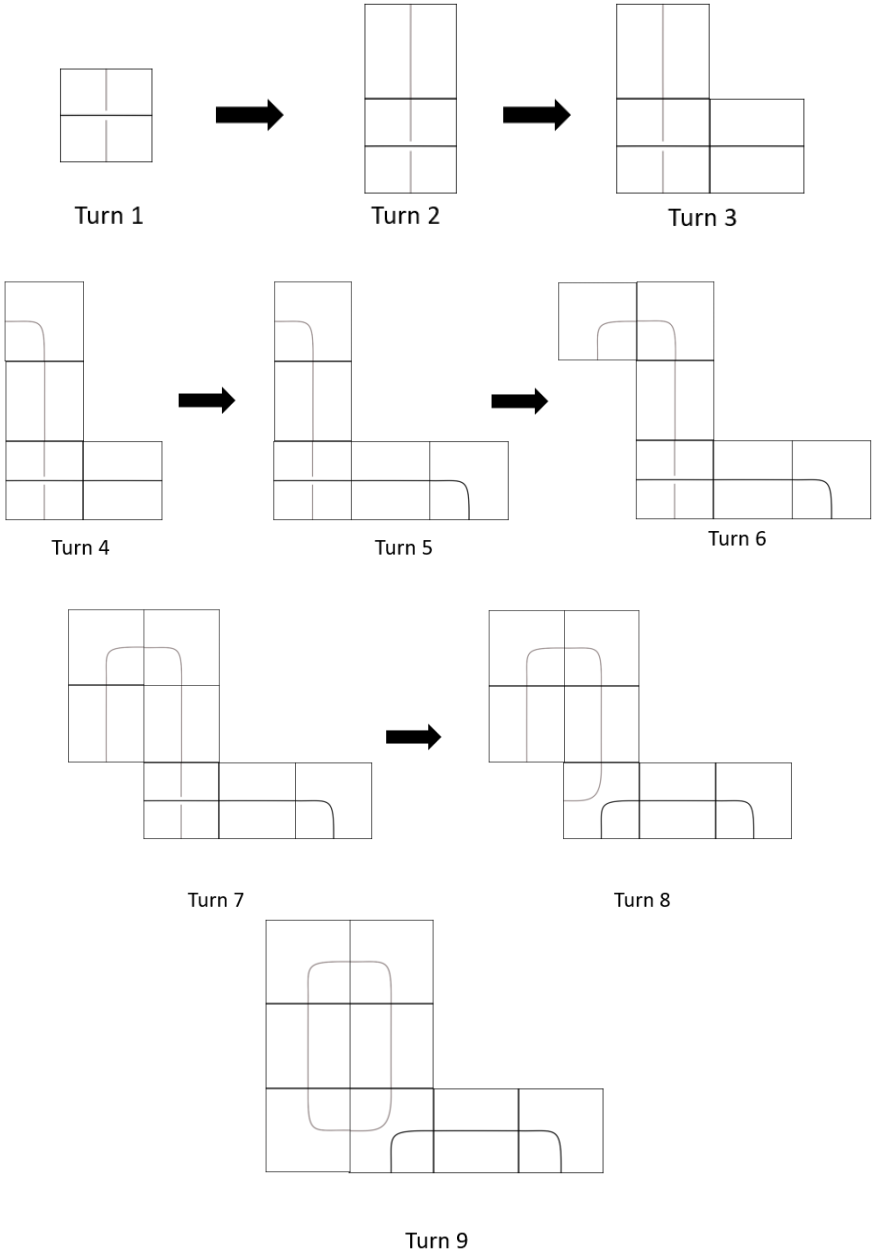

Figure 2.5: A sample game played with the Closed Knot rule set

Figure 2.5 shows a sample run-through of a Closed Knot game. The game begins with both players branching out away from one another in order to build a loop to try and win the game as soon as possible. At turn 8, player 1 decides to create the top part of his loop, but this allows player 2 to close their knot and win the game. If player 1 decided to block player 2 from winning so quickly by placing another piece, the game would have kept going until a closed knot was created.

The main goal of the Closed Knot game is not to highlight any specific mechanics or features of the knot mosaic system, but instead to present the system in a competitive game based on critical thinking and planning. Due to needing a closed knot of a minimum length or width, there is ample time for both players to strategize, plan their future moves, and react to what their opponent is doing all at the same time. In order to win in this game when both players are playing optimally, one must try to set up transformations on the board that the other player will not be able to counteract by their own moves. This leads the Closed Knot game to be one of the most open-ended and strategic games described.

Chapter 3: Playtesting and Feedback

3.1 The Method behind the Testing

In order to test the effectiveness of the game scenarios presented, various playtesting sessions were held. There were three major playtesting sessions held during January and February of 2019, during which the players involved were tasked with playing the four game scenarios described in this paper. The players involved in the playtesting were taken from several different educational backgrounds, including both undergraduate and graduate college students as well as Computer Science, Modeling, and Animation majors, in order to gauge how relevant the topic may have been to them and how easily they could pick up and learn the game rules and mechanics. There will be three major areas of examination when it comes to the effectiveness of these games: mechanics, ease of learning and playing, and fun factor. In addition to these areas of examination, general thoughts and ideas that the players had for improvements and feedback will be included. All of these categories combined will showcase the effectiveness of the games' ability to teach about the knot mosaic system and how enjoyable the games are to play at the same time.

3.2 Examining Game Mechanics

The first area to examine is that of the game mechanics, which includes the moves that can be done each turn, the effectiveness and usability of the special moves, the rule sets for each game, and more. By examining player feedback on the game mechanics and rules, it is possible to see what works and what does not work when put into an actual game, as well as seeing where the game rules could be updated or changed in order to create more balanced or enjoyable game scenarios.

The special moves, which consist of the smoothing technique and the Reidemeister moves, were generally well received by the players. The smoothing technique in particular was used heavily due to its low setup time, which allows either player to utilize the move without needing to have the knot mosaic special moves memorized right after being introduced to them. This also transferred over to the Reidemeister 1 move, which does the same transformation as the smoothing technique but with move setup required to utilize the move. Despite learning about the setup from the Reidemeister 1 move, however, the Reidemeister 2 move did not see much usage in the games due to its higher setup time, the specific setups that the move requires in order to use it, and the lack of necessity when given certain target mosaics. These special moves were given more importance in the Point game scenario, but even then the Reidemeister 2 move did not see much usage due to the other moves giving a decent amount of points with less setup, as well as the victory points enticing players to win more quickly by giving more points than any single special move. In order to fix this issue, the Reidemeister 2 move would need to be given more power in the Point game scenario in order to justify its use alongside the other point sources, as well as creating target mosaics for the other games that encourage the players to use a Reidemeister 2 move based on how the mosaics are set up. Overall, the special moves were very well received and helped to play a big part in every game scenario, albeit some moves more than others.

The nondeterministic tiles ended up serving more of a role as parts of the target mosaics to allow for more creative solutions rather than as pieces that players willingly placed onto the board themselves. While the nondeterministic pieces allow

the players to place pieces that let them make future modifications, it seems that players opted to follow their target mosaics closely instead of playing with more variable pieces like the nondeterministic tiles. The nondeterministic tile property did see more use in the games that used different colored ropes, but this resulted from nearly all tiles implicitly using nondeterministic notation. In order to utilize these tiles in the games that use a single color of rope, there would need to be some incentive to placing those types of tiles on the board, which could be implemented easily in the Point game scenario by giving some point value to placing or using nondeterministic tiles in some way. Overall, the players enjoyed having the freedom of the nondeterministic tiles in their target mosaics, but were not inclined to use these tiles themselves on the board with the given rule sets.

Initially, the multi-colored tiles were not well received when the players were introduced to the concept. However, after playing with the tiles and making use of their unique differences from the single color tiles, the players found that the multi-colored tiles were fun to play with, especially due to the implied nondeterministic tiles. The reception to the tiles improved even more when the players were tasked with playing the Closed Knot game where the tiles could be used to block the players from finishing their closed knot mosaics. Overall, the multi-colored tiles were well received after the players played with them, despite the lukewarm to negative reception when the players were first introduced to the tiles.

Lastly, the rule sets were well received due to being fairly straightforward with easy to understand winning conditions. The most complicated rule sets, which were those for the Point and Colored Rope games, had some extra additions that the

players needed to be reminded of during the playtesting, but overall were also easily picked up after some adjustment time. However, in the Point game scenario, the players did not always think that the point values did not matter at some points since completing a target mosaic gave so many points compared to the other moves that also give points to the players. Overall, the rule sets and special cases for certain game scenarios were picked up by the players decently well, despite needing some reminders while learning the games.

3.3 Examining the Ease of Learning the Games

The second area to examine is that of the games being able to effectively teach the players about the knot mosaic tile system. The main goal of these game scenarios is to teach the players about the knot mosaic system, so it is crucial to know how easily the players are able to pick up and use the material that they are introduced to with these games. Each player played a total of four games, which consisted of one game of the Create the Picture scenario, one game of the Point scenario, one game of the Colored Rope scenario, and one game of the Closed Knot scenario. The games were played in the aforementioned order as well in order to introduce mechanics step by step instead of all at once in an attempt to teach the players more effectively. For the three games that utilize target mosaics, the players were given the target mosaics that were described in each example in this paper but were not told what their opponent's mosaic looked like.

Before playing any games, each player was briefed on the knot mosaic system, the special moves that can be used, and the general game rules that must be followed. The first game, which is the Create the Picture game scenario, serves as the most

basic of the games, with only one rope color and having the players guided by the given target mosaics. The general strategy from the players was to build their target mosaics without much other thought, while using a smoothing technique to change some of the pieces if needed. Moving onto the Point game scenario changed the players' thoughts on how to approach the game since they were no longer able to just create their target mosaics to win. This game saw much heavier use of the special moves in order to gain points, with some even placing more tiles than necessary just to get more points. The Colored Rope game introduced a bit of a hurdle with introducing a brand new rope color and new specific rules for the new rope color, but after playing the first two games the players were able to easily pick up this new tile notation and continue playing in the same way, albeit dealing with the added rope color. Finally, the Closed Knot game saw the most competitive games with some matches lasting for quite some time before having a declared winner. By introducing this game last, the players are able to take all of the knowledge that they have been accumulating over the past three games and use them instinctively to create a closed knot mosaic. Overall, by playing the games in this specific order, the games do a very good job at teaching the players about the knot mosaic system and the mosaic special moves, which naturally see more usage the longer the play testers play the games.

3.4 Examining the Fun Factor of the Games

The final area to examine is that of the games' fun factor. While the main goal of these games is to be educational, it is important to examine how fun the games are to play as having more enjoyable games can increase interest in the topic and encourage players to play multiple times.

Due to placing tiles on the board to set up moves and examining how the opponent is playing, the knot mosaic games are best described as a puzzle strategy game. Players who regularly play other games that require critical thinking and planning, such as Chess, found the games to be more enjoyable than players who do not play such games. Even though this is the case, the players who were less experienced with puzzle and strategy games found the games interesting and enjoyable. Despite the bias towards fans of puzzle and strategy games, everyone that played the games said that they enjoyed it and that there was a level of depth and planning that they did not expect going into playtesting.

3.5 General Player Feedback

Even with the three major examination areas covered, there was some player feedback that does not directly fall into the three categories mentioned. This feedback helps to see what general thoughts players had while playtesting the games and what could potentially be implemented in the future to further improve the games.

The size of the target mosaics was generally favored to be around three tiles vertically or horizontally. Any target mosaics that would be smaller than this in both directions were thought to be too small to work with and did not give enough time to set up any kind of advanced strategies. Similarly, if the target mosaics were larger than 3 tiles vertically or horizontally, then there would be a lot of planning to build the mosaics and messing up a move could potentially require that the entire mosaic be rebuilt. Finding an optimal amount of tiles for the target mosaics ended up being very important to the enjoyment of the games, as this allows the games to allow for players planning moves while also making the games last a good amount of time.

One point of contention during the games was the orientation of the board. In the playtesting, the players sat across from one another while playing, which made some pieces appear differently to the players. While orientation is not a natural problem in the knot mosaic system, having the players see different rotations of the board from their perspectives caused some confusion. In order to alleviate this issue, the rules can be changed such that the players are required to face the same direction. This change is targeted at player enjoyment and knowledge of the board rather than knot mosaic mechanics, but it can be easily implemented to make the game experience less confusing overall.

At the time of writing, the pieces that are used for the games scenarios have a single picture on them that defines what mosaic tile they represent. In order to utilize a transformation, the players would have to pick a tile up off the board and replace that tile with a brand new tile or group of tiles. One suggested change is to make the pieces two-sided, with the reverse side having the picture that represents a mosaic tile after a transformation is made. This change would allow for quicker transformations in game without having to constantly switch out pieces on the board for pieces that are not in play.

Chapter 4: Final Thoughts and Conclusion

The goal of this paper was to examine the knot mosaic system created my Samuel Lomonaco and Louis Kauffman and see how effective it could be as a playable game. Several different game scenarios were created that show off the knot mosaic system in general, as well as highlight the nuances of the system such as knot transformations and multiple ropes interacting. The effectiveness of these games was determined by feedback collected by current undergraduate and graduate college students of multiple different majors and backgrounds. The game system as a whole can be declared as both effective at teaching the players about the knot mosaic system and a fun and enjoyable experience. Even with this conclusion drawn, it is still possible to adjust and refine the rules of the game to improve the player experience and even create new game scenarios not described in this paper.

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