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## AN INSTABILITY OF FINITE AMPLITUDE CIRCULARLY POLARIZED ALFVÉN WAVES

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### ABSTRACT

A circularly polarized, large-amplitude Alfvén wave in a plasma with finite plasma  $\beta$  is shown to be unstable. The wave decays by coupling to random density and magnetic fluctuations in the ambient medium. Daughter waves are produced at the sidebands which are at the first harmonic of the Alfvén wave frequency and wavenumber. This is a four-wave coupling in which the initial Alfvén wave excites forward-propagating density and magnetic waves as well as a backward-propagating magnetic wave. In general, the driven waves are not normal modes of the plasma, although in the limit of  $\beta \rightarrow 0$  the "decay" instability is recovered in which the density wave is ion-acoustic and one of the magnetic waves is a backward-propagating Alfvén wave. For parameters more typical of the solar corona and solar wind, large decay rates are found for  $0.1 \leq \beta \leq 1$  and  $0.1 \leq \eta \leq 0.9$ , where  $\eta$  is the ratio of magnetic energy density of the initial Alfvén wave to that of the background magnetic field. The possible role played by this instability in several astrophysical situations is briefly considered.

*Subject headings:* hydromagnetics — plasmas — polarization — Sun: corona — Sun: solar wind

### 1. INTRODUCTION

Alfvén waves are thought to permeate many astrophysical plasmas. They have been directly observed in the solar wind (Coleman 1966; Unti and Neugebauer 1968; Belcher and Davis 1971), and are expected to exist in the interstellar medium on many length scales and in many environments. They are of great theoretical interest due to their ubiquity and because they are a normal mode in magnetohydrodynamics (MHD) which can be easily studied in the large-amplitude regime. Of great interest during recent years has been the study of the stability of these waves. To first order in the strength of the fluctuating field the waves are stable; but there are several mechanisms known to produce decay of such waves, due either to mode-coupling effects such as the decay instability (Galeev and Oraevskii 1962), or to higher order particle interactions with linearly polarized waves (Hollweg 1971). The decay instability is restricted to the limit of  $\beta \rightarrow 0$  (where  $\beta$  is the ratio of thermal to magnetic energy densities) and thus is probably of no importance in either the solar wind ( $\beta \approx \frac{1}{2}$ ) or the solar envelope ( $\beta \approx 0.1$ ) (Burlaga 1971). (We follow Burlaga [1971] in defining the solar envelope as the region between  $2 R_{\odot}$  and  $25 R_{\odot}$  in the solar atmosphere.) In a somewhat more general framework, Cohen and Dewar (1974) and Cohen (1975) considered a mode-coupling interaction in which it was assumed both that the pump waves were randomly distributed in phase, and that ion-acoustic waves were heavily damped. Although their formalism allowed for finite  $\beta$ , their discussion was by and large confined to the limit of  $\beta \rightarrow 0$ . Other possibilities for instabilities have been considered, among them the work of Chin and Wentzel (1972), which postulated that two of the interacting waves were Alfvénic. In addition, Valley (1974) has considered the role of density fluctuations in scattering and modifying Alfvén waves. In a recent paper, Lashmore-Davies (1976) derived a general modulational instability in which a linearly polarized Alfvén wave coupled to random fluctuations in magnetic field and density. The properties of the resulting dispersion relation were considered in the limit of  $\beta \rightarrow 0$  and  $\omega_r/\omega_0 \rightarrow 0$ , where  $\omega_r$  and  $\omega_0$  are the (real) frequencies of the driven sound wave and pump Alfvén wave, respectively. In general, the three driven waves need not include an Alfvén wave. The decay instability was recovered as a special case, though the modulational instability operated over a wider range of parameters. The work of Lashmore-Davies provides the motivation for the remainder of our discussion. First we argue that circular polarization is a more correct one to consider for the initial Alfvén wave. This leads to a significant simplification of the linear dispersion relation. We then go on to examine the dispersion relation for  $\beta \lesssim 1$  and find several interesting results; among them are the fact that  $\omega_r/\omega_0$  is now not small, so that the density fluctuation has a finite oscillation period even in the absence of dissipation, unless  $\beta \rightarrow 0$ . Furthermore, the decay rate of the Alfvén wave,  $\gamma$ , is very large—of order  $\omega_r$  for  $\beta \lesssim 0.5$ . The role of this mechanism in the solar wind when  $\beta \gtrsim 0.5$ , and consequently the instabilities are not as strong, is not yet clear. However, in situations where  $\beta \lesssim 0.5$ , as it is in the solar envelope and at times in the solar wind, we conclude that finite-amplitude, circularly polarized Alfvén waves are generally unstable.

## II. LINEAR DISPERSION RELATION

Our starting point is the set of MHD equations written in the form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla (P + B^2/8\pi) + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B}, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{B} \nabla \cdot \mathbf{v} + (\mathbf{B} \cdot \nabla) \mathbf{v} - (\mathbf{v} \cdot \nabla) \mathbf{B}. \quad (3)$$

For convenience the dissipation terms produced by finite resistivity, viscosity, and collisions are not included as they only serve to slightly complicate the subsequent analysis without altering the basic physics. In addition, we assume an isothermal equation of state,  $P = \rho c_s^2$ , where  $c_s$  is the sound speed.

It is well known that these equations permit the existence of large-amplitude Alfvén waves with the following properties (Hollweg 1974; Barnes and Hollweg 1974; Goldstein, Klimas, and Barish 1974; Barnes 1976): (a) the fluid flow is incompressible ( $\nabla \cdot \mathbf{v} = 0$ ); (b) the magnitude of the magnetic field is a constant; (c) the fluctuations in fluid velocity and magnetic field are related by  $\mathbf{v}' = \pm \mathbf{B}'/(4\pi\rho)^{1/2}$ ; and (d) the wave structure moves along the mean field,  $\mathbf{B}_0$  with velocity  $c_A = \pm B_0/(4\pi\rho)^{1/2}$ . Ultimately one would like to investigate the stability properties of such disturbances, as there is some recent evidence that interplanetary fluctuations occasionally exhibit some of these features (Sari 1977). However, in this communication we will have to restrict our attention to large-amplitude waves that are planar with magnetic fluctuations transverse to  $\mathbf{B}_0$ . Thus the large-amplitude waves whose stability we investigate will satisfy  $\mathbf{v}_\perp = \pm \mathbf{B}_\perp/(4\pi\rho)^{1/2}$ , and all quantities will be assumed to vary only along  $\mathbf{B}_0$ . This limitation to a “slab” geometry is the most restrictive one we make, and the most difficult to relax.

The pump wave will be a single, monochromatic, large-amplitude Alfvén wave, which we take to have circular polarization in order to maintain constant field magnitude. The implications resulting from this choice of polarization will become clear below. We now further assume that in addition to the pump wave the fluid contains small random fluctuations. (It is easy to show that  $B'_z = B'_\parallel$  must be zero.) Thus we can write  $\rho(z, t) = \rho_0 + \rho'(z, t)$ ,  $\mathbf{v}(z, t) = \mathbf{v}_\perp(z, t) + \mathbf{v}_\parallel(z, t) + v_\parallel'(z, t)$ ,  $\mathbf{B}(z, t) = B_0\hat{e}_z + \mathbf{B}_\perp(z, t) + \mathbf{B}_\perp'(z, t)$ , where  $\mathbf{v}_\perp(z, t) = \pm \mathbf{B}_\perp(z, t)/(4\pi\rho_0)^{1/2}$  characterizes the initial Alfvén wave, and  $\rho'(z, t)$ ,  $v_\parallel'(z, t)$ ,  $\mathbf{B}_\perp'(z, t)$  characterize the small fluid fluctuations. (Note that  $\mathbf{B}_\perp$  is not assumed small compared to  $\mathbf{B}_0$ .) We do not need to assume that any of the random fluctuations are Alfvénic. One immediately finds that  $\rho'$  and  $\mathbf{B}_\perp'$  satisfy the following driven wave equations:

$$\frac{\partial^2 \rho'}{\partial t^2} - c_s^2 \frac{\partial^2 \rho'}{\partial z^2} = \frac{1}{4\pi} \frac{\partial^2}{\partial z^2} (\mathbf{B}_\perp \cdot \mathbf{B}_\perp') \quad (4)$$

$$\frac{\partial^2 \mathbf{B}_\perp'}{\partial t^2} - c_A^2 \frac{\partial^2}{\partial z^2} \mathbf{B}_\perp' = -\frac{\partial^2}{\partial z \partial t} (v_\parallel' \mathbf{B}_\perp) - B_0 \frac{\partial}{\partial z} \left( v_\parallel' \frac{\partial}{\partial z} \mathbf{v}_\perp \right) - \frac{B_0}{\rho_0} \frac{\partial}{\partial z} \left( \rho' \frac{\partial}{\partial t} \mathbf{v}_\perp \right), \quad (5)$$

in which terms on the right side that are proportional to products of small fluctuations have been dropped.

Equations (4) and (5) were previously derived by Lashmore-Davies (1976) under the assumption that the pump wave was linearly polarized, which is not possible if the wave is incompressible. Hollweg (1971), also working with a linearly polarized, but compressive, wave, found an equation similar to (4) in which the right side was proportional to  $\partial^2 (\mathbf{B}_\perp \cdot \mathbf{B}_\perp)/\partial z^2$ . Such a term is identically zero for the noncompressive wave we have employed.

If we now define the unit vector for circular polarization as  $\hat{e}_\pm \equiv (\hat{e} \pm i\hat{e}_y)/\sqrt{2}$ , in which the plus and minus denote left and right circular polarization, respectively, the pump and daughter magnetic fluctuations can be written as

$$\mathbf{B}_\perp(z, t) = \frac{1}{2} \{ B \exp[-i(k_0 z - \omega_0 t)] \hat{e}_\pm + B^* \exp[i(k_0 z - \omega_0 t)] \hat{e}_\pm^* \},$$

$$\mathbf{B}_\perp'(z, t) = \frac{1}{2} [B'(z, t) \hat{e}_\pm + B'^*(z, t) \hat{e}_\pm^*]. \quad (6)$$

From equation (4) it is clear that only fluctuations with a sense of polarization (or helicity) equal to that of the pump wave can be excited in this interaction.

A dispersion relation can be found after one uses the zero-order expansions of equations (2) and (3) to remove  $\mathbf{v}_\perp$  from equation (5). Then the Fourier spacetime transforms of equations (4) and (5) lead to

$$(-\omega^2 + k^2 c_s^2) \rho'(k, \omega) = \frac{-k^2}{8\pi} [BB'^*(-k_-, -\omega_-) + B^*B'(k_+, \omega_+)] \quad (7)$$

and

$$\begin{aligned} (-\omega_+^2 + k_+^2 c_A^2) B'(k_+, \omega_+) &= \frac{k_+ \omega_0^2 B}{2k_0} \left[ 1 - \left( \frac{\omega}{k} \right) \left( \frac{k_0}{\omega_0} \right) - \frac{\omega_+}{\omega_0} \left( \frac{\omega}{k} \right) \left( \frac{k_0}{\omega_0} \right) \right] \frac{\rho'(k, \omega)}{\rho_0}, \\ (-\omega_-^2 + k_-^2 c_A^2) B'^*(-k_-, -\omega_-) &= \frac{-k_- \omega_0^2 B^*}{2k_0} \left[ 1 - \left( \frac{\omega}{k} \right) \left( \frac{k_0}{\omega_0} \right) + \frac{\omega_-}{\omega_0} \left( \frac{\omega}{k} \right) \left( \frac{k_0}{\omega_0} \right) \right] \frac{\rho'(k, \omega)}{\rho_0}, \end{aligned} \quad (8)$$

in which  $k_{\pm} \equiv k \pm k_0$  and  $\omega_{\pm} \equiv \omega \pm \omega_0$ . Equations (7) and (8) are exact at the linear level. Coupling to higher order sidebands does not appear if the pump wave is circularly polarized. When equations (8) are substituted into equation (7), the exact linear dispersion relation for complex frequency  $\omega$  can be rewritten in the dimensionless form

$$(\omega^2 - \beta k^2)(\omega - k)[(\omega + k)^2 - 4] = \frac{\eta k^2}{2} (\omega^3 + \omega^2 k - 3\omega + k), \quad (9)$$

where from now on frequencies and wavenumbers will be normalized to those of the pump wave (i.e.,  $\omega/\omega_0 \rightarrow \omega$  and  $k/k_0 \rightarrow k$ ). We have also defined  $\eta \equiv |B|^2/B_0^2$  and  $\beta \equiv c_s^2/c_A^2$ .

The usual procedure (Lashmore-Davies 1976) is to let  $\beta \rightarrow 0$  and look for solutions for which  $\text{Re}(\omega) \ll 1$ . We shall see below that this yields very misleading (if not incorrect) results, and the fact that  $0.1 < \beta \lesssim 1$  in the solar corona and solar wind motivates us to examine equation (9) as it stands.

The first striking thing about equation (9) is that the singularity at  $k = \pm 2$  that arises in the  $\beta \rightarrow 0$  limit (Lashmore-Davies 1976) is absent here. The second prominent feature is that equation (9) is fifth-order in  $\omega$ , which precludes analytic solution. One can use Sturm sequences (e.g., Burnside and Panton 1960) to determine criteria under which the dispersion relation has complex solutions ( $\omega \equiv \omega_r + i\gamma$ ) and thus explicitly exhibit instability thresholds as functions of  $\eta$  and  $\beta$ . However, the complexity of the resulting algebraic formulas requires graphical presentation, and so we find it more illuminating to present representative numerical solutions of equation (9) for parameters typical of the solar envelope and solar wind. One should note that  $|\gamma|$  is both the decay rate of the pump wave and the growth rate of the daughter waves.

In Figure 1 plots of  $|\gamma(k)|$  are shown versus  $k$  for  $\beta = 0.1$  and  $0.55$ , and for two values of  $\eta$ . The decay rate of the Alfvén wave is seen to be substantial even for  $\beta = 0.55$  and  $\eta = 0.9$ . In fact, for  $\beta$  as large as  $1.0$ , Alfvén waves are unstable with  $\gamma = 0.012$  at  $k = 1.05$  and  $\eta = 0.1$ . As one might expect, larger  $\eta$  produces higher values of  $|\gamma|$ , whereas, as  $\beta$  increases, the fluid becomes less compressive and density waves are more difficult to excite. This causes both  $|\gamma|$  and the range of unstable wavenumbers to become smaller as is illustrated in Figure 2 where the maximum value of  $\gamma(k)$  is plotted versus  $\eta$  for  $\beta = 0.1$  and  $0.9$  in Figure 2a, and versus  $\beta$  for  $\eta = 0.9$  and  $0.1$  in Figure 2b.

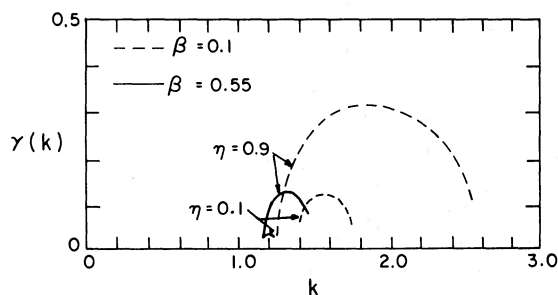


FIG. 1

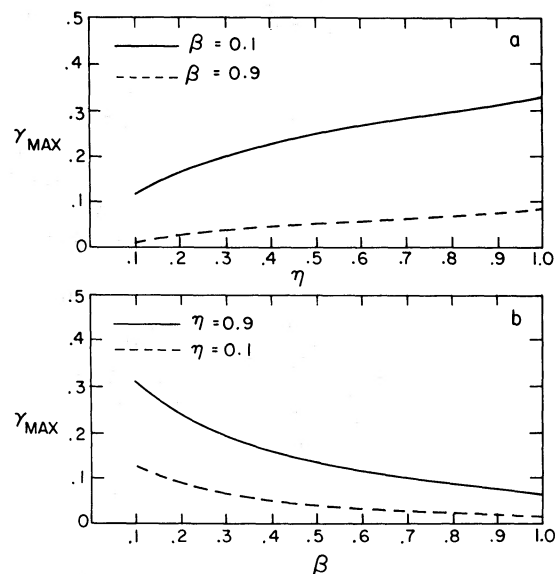


FIG. 2

FIG. 1.—The decay rate of the pump Alfvén waves,  $\gamma(k)$ , versus wavenumber for  $\beta = 0.1$  and  $0.55$  and  $\eta = 0.1$  and  $0.9$ .  
FIG. 2.—The maximum value of  $\gamma(k)$  against  $\eta$  for  $\beta = 0.1$  and  $0.9$  in Fig. 2a and against  $\beta$  for  $\eta = 0.1$  and  $0.9$  in Fig. 2b.

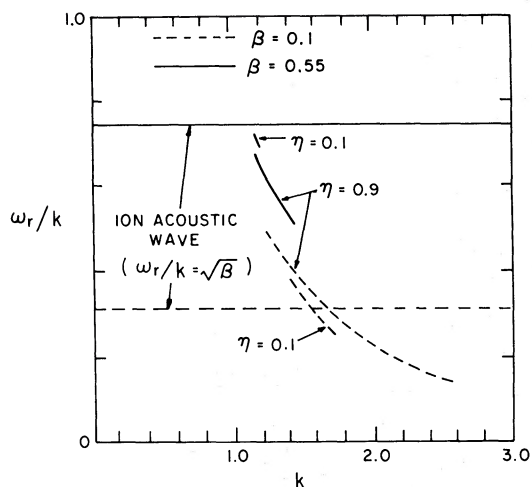


FIG. 3

FIG. 3.—Phase velocity of the ion fluctuations plotted against wavenumber for the same parameters as in Fig. 1. An ion acoustic wave would satisfy  $\omega_r/k = \sqrt{\beta}$ .

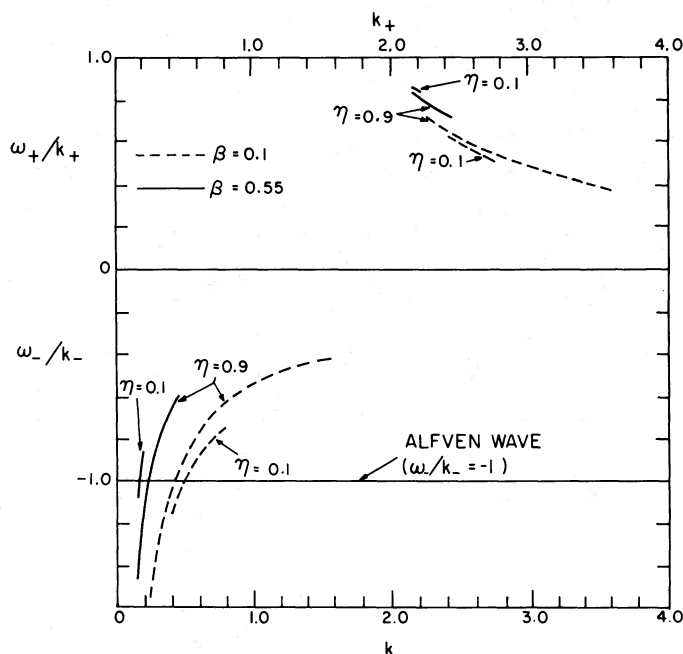


FIG. 4

FIG. 4.—Phase velocities of the upper and lower sidebands plotted against wavenumber. Alfvén waves would satisfy  $\omega_+/k_+ = 1$  or  $\omega_-/k_- = -1$ .

In general, the unstable waves at  $\omega_r$  and  $\omega_{\pm}$  are not normal modes of the plasma, i.e., the density fluctuations excited at  $\omega_r(k)$  are not ion-acoustic, nor are the magnetic fluctuations at  $\omega_{\pm}(k_{\pm})$  Alfvén waves. This is most easily seen by plotting  $\omega_r/k$  and  $\omega_{\pm}/k_{\pm}$  versus  $k$  and comparing with the ion-acoustic ( $\omega_r/k = \sqrt{\beta}$ ) and Alfvén ( $|\omega_{\pm}/k_{\pm}| = 1$ ) dispersion relations. These are shown in Figures 3 and 4 for  $\beta = 0.1$  and  $0.55$ . The larger  $\beta$  the greater the divergence of the driven waves from the normal mode dispersion relations. However, there always appears to be one value of  $k_-$  for which the lower sideband wave has a phase velocity equal to the Alfvén velocity. This value of  $k_-$  occurs near the maximum of  $\gamma(k)$ , especially for  $\eta \approx 0.1$ . Thus, if this modulational instability drives a turbulent cascade (a question to which we return below), one might expect that in the inverse cascade region ( $k < 1$ ) the magnetic fluctuations would closely resemble Alfvén waves, while in the inertial range ( $k > 1$ ) the wave would possess more complicated dispersion properties. (See Montgomery 1976 for a recent review of turbulence theory for MHD fluids.)

The well-known results for the decay instability (e.g., Sagdeev and Galeev 1969, pp. 8–11), can be recovered from this dispersion relation in the limit of  $\beta \ll 1$ . In that case the solution of equation (9) yields  $\omega_r \ll 1$  with  $\gamma(k)$  reaching a maximum near  $k \approx 2$ . Furthermore, in the vicinity of  $k = 2$ ,  $\omega_-/k_- \approx -1$ . Thus, in very low- $\beta$  plasmas we recover the result that an Alfvén wave decays into a forward-moving ion acoustic wave and a backward-moving Alfvén wave. The upper sideband wave is also excited, but since it is not a normal mode its amplitude is expected to be small (Lashmore-Davies 1976). In the limit of  $\beta \rightarrow 0$ , the decay rate will contain a singularity at  $k = \pm 2$  which can be removed with the inclusion of dissipation. It is only in this limit that equation (11) of Lashmore-Davies (1976) has validity.

### III. DISCUSSION

We have shown that a finite-amplitude, circularly polarized Alfvén wave is, in general, unstable in a MHD fluid. This is a four-wave coupling process in which the daughter waves are two forward-propagating waves—one in density, the other in magnetic field—and one backward propagating magnetic wave. Furthermore, the magnetic waves preserve the helicity of the pump wave. This general coupling has not previously been investigated in connection with questions of the stability of such Alfvén waves; thus the oft-quoted assertion (e.g., Hollweg 1975) that such waves are stable. It is worth emphasizing that this modulational instability requires the existence of small random fluctuations in density and magnetic field—a requirement we think easily satisfied in any astrophysical plasma.

In the remaining discussion we call attention to a few astrophysical situations in which this instability may play a significant role: Belcher and Davis (1971) have conjectured that the observation of predominately outward-



propagating Alfvén waves in the solar wind indicates that the waves are produced near the Sun and are then convected outward into the solar wind. Hollweg (1972) and others have suggested the supergranulation as the source of these waves. A subsequent paper (Hollweg 1973) considers the effect of such a source of Alfvén waves on a two-fluid model of the solar wind in which it is assumed that there exists an unspecified instability which restricts  $\eta$  to values  $\leq \frac{1}{2}$  in the solar envelope. The modulational instability we have described could in fact provide the mechanism for limiting the value of  $\eta$ , though as we have seen, even  $\eta = 0.1$  produces a significant decay rate which may imply a deposition of energy into the corona at somewhat lower altitudes than was originally considered. The concomitant effects on solar wind parameters at 1 AU remain to be investigated.

The role of this instability in the solar wind where large-amplitude Alfvénic disturbance are often observed (e.g., Abraham-Shrauner and Feldman 1977; Burlaga and Denskat 1977) is not clear, for in the solar wind the decay rates are smaller than in the corona ( $|\gamma| \lesssim 0.1$ ), and wave-particle damping of the driven density fluctuations may be significant. However, there are recent solar wind observations reported by Burlaga and Turner (1976) that can be interpreted as evidence that this mode-coupling may be operating. From a study of 1 hour plots of 15 s averages of magnetic field data, those authors noted the existence of a class of magnetic fluctuations with characteristics of large-amplitude "Alfvén" waves propagating outward from the Sun nearly along  $B_0$  and which exhibited nonzero fluctuations in the field magnitude, i.e.,  $\delta B/B \approx 0.06$ . Thus those fluctuations could not be pure transverse Alfvén waves. A possible interpretation of that observation can be constructed from our discussions above. Because the magnetic fluctuations arising from this instability all have circular polarization, the maximum change in the magnitude of  $B$  is

$$\delta B \approx [2|B_{\perp}| \cdot |B_{\perp}'|]^{1/2} \quad (10)$$

or

$$\frac{\delta B}{|B|} \approx \left[ 2 \left( \frac{B_{\perp}}{B} \right)^2 \left( \frac{B_{\perp}'}{B_{\perp}} \right) \right]^{1/2} \approx 0.06$$

if one uses  $(B_{\perp}/B) \approx 0.3$  and  $B_{\perp}'/B_{\perp} \approx 0.02$ . Thus even a small contribution from this instability would suffice to produce the observed variation in the magnitude of  $B$ . For  $|B_{\perp}|/|B| \approx 0.3$  and  $\beta \approx \frac{1}{2}$ , the decay rate of a circularly polarized Alfvén wave in the solar wind is not large,  $\gamma \approx 0.03$ . However, if one takes the wave period as 10 min, then, even including a large Doppler shift between the reference frame of the spacecraft and the solar wind, amplitudes of the daughter waves can grow by a factor of  $e$  within  $\sim 0.1$  AU. Whether this is adequate to provide a quantitative explanation of Burlaga and Turner's observations requires a more detailed analysis than can be given here.

Another fundamental question that remains to be examined is whether this instability will produce a turbulent cascade. Because the daughter waves are not in general pure Alfvén and ion acoustic waves, the question is not answerable at the level of analysis given here. However, much of the ultimate importance of this modulational instability in astrophysical plasmas hinges on an affirmative answer to that question. A detailed investigation of this point is currently under way.

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#### REFERENCES

- Abraham-Shrauner, B., and Feldman, W. C. 1977, *J. Geophys. Res.*, **82**, 618.  
 Barnes, A. 1976, *J. Geophys. Res.*, **81**, 281.  
 Barnes, A., and Hollweg, J. V. 1974, *J. Geophys. Res.*, **79**, 2302.  
 Belcher, J. W., and Davis, L., Jr. 1971, *J. Geophys. Res.*, **76**, 3534.  
 Burlaga, L. F. 1971, NASA/GSFC Tech. Rept. X692-71-400.  
 Burlaga, L. F., and Turner, J. M. 1976, *J. Geophys. Res.*, **81**, 73.  
 Burnside, W. S., and Panton, A. W. 1960, *The Theory of Equations*, vol. 1 (New York: Dover), pp. 198 ff.  
 Chin, Y.-C., and Wentzel, D. G. 1972, *Ap. Space Sci.*, **16**, 465.  
 Cohen, R. H. 1975, *J. Geophys. Res.*, **80**, 3678.  
 Cohen, R. H., and Dewar, R. L. 1974, *J. Geophys. Res.*, **79**, 4174.  
 Coleman, P. J., Jr. 1966, *Phys. Rev. Letters*, **17**, 201.  
 Denskat, U., and Burlaga, L. F. 1977, *J. Geophys. Res.*, **82**, 2693.  
 Galeev, A. A., and Oraevskii, V. N. 1962, *Dokl. Akad. Nauk. SSR*, **147**, 71 (Engl. trans. in *Soviet Phys.—Dokl.*, **7**, 988 [1963]).  
 Goldstein, M. L., Klimas, A. J., and Barish, F. D. 1974, in *Solar Wind Three*, ed. C. T. Russell (Los Angeles: University of California Press), p. 385.  
 Hollweg, J. V. 1971, *Phys. Rev. Letters*, **27**, 1349.  
 ———. 1972, *Cosmic Electrodyn.*, **2**, 423.  
 ———. 1973, *Ap. J.*, **181**, 547.  
 ———. 1974, *J. Geophys. Res.*, **79**, 1539.  
 ———. 1975, *Rev. Geophys. Space Phys.*, **13**, 263.  
 Lashmore-Davies, C. N. 1976, *Phys. Fluids*, **19**, 587.  
 Montgomery, D. 1976, University of Iowa preprint, 76-33.  
 Sagdeev, R. Z., and Galeev, A. A. 1969, *Nonlinear Plasma Theory* (New York: W. A. Benjamin).  
 Sari, J. W. 1977, NASA/GSFC Tech. Rept. X-692-77-170.  
 Unti, T. W., and Neugebauer, M. 1968, *Phys. Fluids*, **11**, 563.  
 Valley, G. C. 1974, *Ap. J.*, **188**, 181.

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