Analytically parameterized solutions for robust quantum control using smooth pulses

Utkan Güngördü* and J. P. Kestner
Department of Physics, University of Maryland Baltimore County, Baltimore, MD 21250, USA

Achieving high-fidelity control of quantum systems is essential for realization of a practical quantum computer. Composite pulse sequences which suppress different types of errors can be nested to suppress a wide variety of errors but the result is often not optimal, especially in the presence of constraints such as bandwidth limitations. Robust smooth pulse shaping provides flexibility, but obtaining such analytical pulse shapes is a non-trivial problem, and choosing the appropriate parameters typically requires a numerical search in a high-dimensional space. In this work, we extend a previous analytical treatment of robust smooth pulses to allow the determination of pulse parameters without numerical search. We also show that the problem can be reduced to a set of coupled ordinary differential equations which allows for a more streamlined numerical treatment.

arXiv:1906.12289v1 [cond-mat.mes-hall] 28 Jun 2019

I. INTRODUCTION

The main difficulty hampering the efforts to build a large scale, practical quantum computer is decoherence. Quantum error correction codes provide a promising path toward fault-tolerant quantum computers. However, a typical surface code requires access to quantum gates with a fidelity above 99%, and significantly higher fidelities are desirable to reduce overhead. Achieving high gate fidelities in a noisy device requires carefully designed robust control fields.

Robust composite pulse sequences [1], which generalize Hahn echo [2] and Carr-Purcell-Meiboom-Gill (CPMG) [3, 4] sequences to implement non-trivial unitaries, are effective for suppressing slow noise or calibration errors which remain constant during the gate time. Various pulse sequences have been developed to suppress either pulse length errors or off-resonance errors [1]. However, in some systems, such as spin qubits in silicon [5–9] or GaAs [10, 11], noise is present in some combination of the two forms, which requires nesting these sequences [12] or using specialized pulses [13–15]. Such methods are often designed with square pulses in mind, although they can be modified to use smooth ramping profiles [14, 16]. However, the finite bandwidth of a physical control field may be more naturally accommodated by robust smooth pulses [17–20]. These smooth pulses have an analytical form, but with free parameters that must be chosen to produce the desired unitary while satisfying robustness constraints, and this usually requires a numerical search in parameter space.

In this paper, based on the approach of Ref. [18], we derive a completely analytical family of robust smooth pulses which eliminates the requirement of numerical parameter fitting. We also cast the problem of finding a robust smooth pulse which implements a particular unitary into a set of coupled ordinary differential equations (ODEs), which can be solved by using standard numerical solvers. We provide explicit examples of robust pulse shapes along with their filter functions.

* utkan@umbc.edu

Although our focus here will be on a two-level system, the physical context is not necessarily limited to one-qubit problems. Indeed, these solutions can be used to implement robust gates in SU(2) $\subset$ SU(4) or SU(2)$\times$SU(2) $\subset$ SU(4) subgroups, targeting local rotations or non-local controlled-phase gates in a silicon double quantum dot setup [16] or in superconducting qubits with fixed coupling [21].

The structure of this paper is as follows. In Sec. II, we present a brief summary of the analytical formalism of Barnes et al. [18] on which this work is built. In Sec. III, we show how to choose symmetric auxilliary functions and their parameters without resorting to a numerical search, and we present the resulting pulse shapes and filter functions. In Sec. IV, we show how to efficiently generate robust pulse shapes by introducing auxilliary ODEs and incorporating the desired rotation angles and robustness constraints as local boundary conditions rather than nonlocal integral relations. We then conclude in Sec. V.

II. BACKGROUND

We first review robust smooth pulses for a one-qubit system, adapted from Ref. [18] to our use cases. We consider the two-level Hamiltonian

$$\tilde{H} = \Omega(t)\sigma_x + \tilde{\beta}\sigma_z, \tag{1}$$

where $\Omega(t)$ represents the driving field, and $\tilde{\beta} = \beta + \delta\beta$ is the energy splitting with non-Markovian fluctuations $\delta\beta$. This Hamiltonian appears in various systems including solid state spin qubits. In the absence of noise, the time evolution operator $U(t_f; 0)$ at $t = t_f$ can be parametrized in terms of an auxilliary function Φ in the following way [18]:

$$U(t_f) = X_{\xi_-(\chi_f)-\xi_+(\chi_f)} Z_{2\chi_f} X_{\xi_-(\chi_f)+\xi_+(\chi_f)} \tag{2}$$

where X_γ (Z_γ) denotes a rotation around the $x-$ ($z-$) axis of the Bloch sphere by angle γ and $\xi_\pm$ is related to the control field Ω through the relationships

$$\xi_\pm(\chi_f) = \Phi(\chi_f) \mp \operatorname{sgn}(\Phi'(\chi_f))\frac{1}{2}\operatorname{arcsec}\left(\sqrt{1 + [\Phi'(\chi_f)\sin(2\chi_f)]^2}\right), \tag{3}$$

$$\sinh v(\chi) \equiv \Phi'(\chi)\sin(2\chi), \tag{4}$$

$$\Omega(t) = \Omega(\chi(t)) = \frac{\beta}{\sin(2\chi)}\partial_\chi[\tanh v(\chi)\sin(2\chi)], \tag{5}$$

and χ is a function of time through the relationship

$$\beta t = \hbar \int_0^\chi d\bar{\chi} \cosh v(\bar{\chi}). \tag{6}$$

We have used the notation $\chi_f \equiv \chi(t_f)$. Note that by choosing the free function Φ appropriately, one can generate a desired evolution operator and solve for the corresponding control field. The initial condition $U(0) = \mathbb{1}$ implies $\Phi(0) = \Phi'(0) = 0$.

The time-evolution becomes robust against quasistatic noise in β when the following conditions are satisfied [18]:

$$\sin(4\chi_f) + 8\int_0^{\chi_f} d\chi \sin^2(2\chi)e^{2i[\Phi(\chi)-\Phi(\chi_f)]} = 0 \tag{7}$$

$$\int_0^{\chi_f} d\chi \sin^2(2\chi)\Phi'(\chi) = 0. \tag{8}$$

The real and imaginary parts of the left hand side of Eq. (7) are proportional to first order variations $\delta_\beta\chi(t_f)$ and $\delta_\beta\dot{\chi}(t_f)$, respectively, and the left hand side of Eq. (8) is $\delta_\beta\xi(t_f)$ [18].

The second condition is automatically satisfied for any driving pulse that is antisymmetric: When $\Phi(\chi)$ is an even function, Eq. (8) is odd in χ_f, so for any choice of $\Phi(\chi)$ on the interval $[0, \chi_f]$, or correspondingly, $\Omega(t)$ on $[0, t_f]$, one can construct a rotation robust against $\delta_\beta\xi(t_f)$ by extending the evolution to the interval $[-t_f, t_f]$ with $\Omega(t)$ at negative times defined by enforcing antisymmetry. One can show that the resulting robust rotation on the Bloch sphere is by an angle $\theta = 4\chi_f$ around an axis $\cos(\phi)\hat{x} + \sin(\phi)\hat{y}$ where [18]

$$\cos\phi = \frac{1}{\sqrt{1 + [\Phi'(\chi_f)\sin(2\chi_f)]^2}}. \tag{9}$$

When the bandwidth on the control field $\Omega(t)$ is limited such that it cannot be turned on or off quickly (when compared to the timescale $\hbar/\Omega(t)$), one can furthermore require that $\Omega(t_f)$ also vanishes, which can be viewed as an additional condition on $\Phi''(\chi_f)$ [18].

The first of the robustness conditions, the complex-valued Eq. (7), however, cannot be as trivially satisfied. When targeting an arbitrary rotation, it is possible to find solutions by starting with an ansatz for the auxiliary function $\Phi(\chi, \boldsymbol{a})$ with sufficient degrees of freedom encapsulated as $\boldsymbol{a}$, and use a numerical search to find a_i which would satisfy the robustness conditions while at the same time producing the desired rotation [18]. For the special case of $\chi_f = n\pi/4$, analytical solutions were given in [18]. In the next section, we show how to satisfy Eq. (7) analytically for an *arbitrary* unitary.

III. ANALYTICAL SOLUTIONS TO THE ONE-QUBIT ROBUSTNESS CONDITIONS

Within this section, we assume that Eq. (8) will be satisfied by doubling the interval to $[-\chi_f, \chi_f]$ and using symmetry as discussed above. Thus, we only need to focus on satisfying Eq. (7). This will ensure that the strength of the leading order noise term in the time-evolution operator vanishes at the final time, which is parametrized as χ_f. We notice that this needs to hold only at the final time, and not necessarily at other times, so we introduce a complex valued function $\epsilon(\chi)$ which has the same functional form as the robustness condition as

$$\tan(2\chi)\epsilon(\chi) \equiv \sin(4\chi)e^{2i\Phi(\chi)} + 8\int_0^\chi d\bar{\chi}\sin^2(2\bar{\chi})e^{2i\Phi(\bar{\chi})}, \tag{10}$$

In terms of $\epsilon(\chi)$, the robustness condition translates into a boundary condition at the final time: $\epsilon(\chi_f) = 0$. By making a variable transformation

$$\mathcal{E}(\chi) \equiv \int_0^\chi d\bar{\chi}\sin^2(2\bar{\chi})e^{2i\Phi(\bar{\chi})}, \tag{11}$$

Eq. (10) can be expressed in a simpler form

$$\mathcal{E}'(\chi) + 4\tan(2\chi)\mathcal{E}(\chi) = \epsilon(\chi)/2 \tag{12}$$

which has the solution

$$\mathcal{E}(\chi) = \cos^2(2\chi)\int_0^\chi d\bar{\chi}\frac{\epsilon(\bar{\chi})}{2\cos^2(2\bar{\chi})} \equiv \mathcal{R}(\chi)e^{i[2\Phi(\chi)-\alpha(\chi)]} \tag{13}$$

where we have defined new variables $\mathcal{R}(\chi)$ and $\alpha(\chi)$ related to the amplitude and phase of the integral. By differentiating both sides of Eq. (13), using Eq. (11), and dividing by $\sin^2(2\chi)$, we obtain

$$e^{i\alpha(\chi)} = \frac{\mathcal{R}'(\chi) + i\mathcal{R}(\chi)[2\Phi'(\chi) - \alpha'(\chi)]}{\sin^2(2\chi)}. \tag{14}$$

For a given $\alpha(\chi)$, solving the real and imaginary parts of Eq. (14) for $\mathcal{R}(\chi)$ and $\Phi(\chi)$, respectively, gives

$$\mathcal{R}(\chi) = \int_0^\chi d\bar{\chi}\cos(\alpha(\bar{\chi}))\sin(2\bar{\chi})^2, \tag{15}$$
$$\Phi'(\chi) = \frac{1}{2}\left[\alpha'(\chi) + \frac{\sin(\alpha(\chi))\sin^2(2\chi)}{\mathcal{R}(\chi)}\right].$$

At this point, we have reparametrized the solution of the Schrödinger equation in terms of a function $\alpha(\chi)$ instead of $\Phi(\chi)$, and showed how it would be related to $\Phi(\chi)$ of the original parameterization. The advantage of this reparametrization is that the robustness conditions simplify into *local* boundary conditions for $\alpha(\chi)$ as opposed to *nonlocal* robustness conditions on $\Phi(\chi)$, as we will show shortly. Furthermore, the problem is analytically solvable when $\alpha(\chi)$ is chosen in such a way that $\cos\alpha(\chi)\sin^2(2\chi)$ is integrable. A better alternative, however, is to consider $\mathcal{R}(\chi)$ as the independent variable, which in turn defines $\alpha(\chi)$ through its derivative.

Before going into the robustness conditions, though, we note that the condition $\Phi(0) = 0$ simply corresponds to a vanishing integration constant in Eq. 15. However, the condition $\Phi'(0) = 0$ requires special care: since the denominator vanishes in the limit $\chi \to 0$ (and possibly at other points, depending on the choice for $\alpha(\chi)$), we impose $\sin(\alpha(\chi)) = 0$ at these points to avoid any singularities. This is a stronger condition than requiring that the strength of the control pulse $|\Omega(\chi)|$ remains finite, but leads to a simpler set of constraints.

Finally, we can state the robustness condition, $\epsilon(\chi_f) = 0$, in terms of $\alpha(\chi)$ and $\mathcal{R}(\chi)$ by noting that

$$\begin{aligned}\mathrm{Re}\left(e^{i\alpha(\chi)-2i\Phi(\chi)}\epsilon(\chi)\right) =& 2[4\mathcal{R}(\chi)\tan(2\chi)+\cos(\alpha(\chi))\sin^2(2\chi)],\\ \mathrm{Im}\left(e^{i\alpha(\chi)-2i\Phi(\chi)}\epsilon(\chi)\right) =& 2\sin(\alpha(\chi))\sin^2(2\chi).\end{aligned} \tag{16}$$

At $\chi = \chi_f$, for a generic value χ_f (recalling that it is one of the Euler angles of the final rotation and hence should not be restricted), these relations reduce to

$$\alpha(\chi_f) = n\pi, \qquad \mathcal{R}(\chi_f) = \frac{(-1)^n}{8}\sin(4\chi_f), \tag{17}$$

where n is any integer.

At this point, we remark that even when we treat $\mathcal{R}(\chi)$ as the parametrizing function of the problem, it cannot be chosen arbitrarily, because Eq. 13 implies that its derivative must be a function within $[-1, 1]$ at all times. Furthermore, the robustness condition Eq. 17 for $\alpha(\chi)$ translates into a boundary condition on $\mathcal{R}'(\chi)/\sin^2(2\chi)$.

The relation between the phase, $\alpha(\chi)$, and the amplitude $\mathcal{R}(\chi)$ can be slightly simplified by introducing a new variable $u(\chi) \equiv 4\chi - \sin(4\chi)$ and define $\beta(u(\chi)) \equiv \alpha(\chi)$, such that $\mathcal{R}(u) = \int_0^u du \cos\beta(u)$. Using $u \equiv u(\chi)$ instead of χ as the "independent" parameter is not essential and working with $\mathcal{R}(\chi)$ is equally possible and will be used in Section IV when finding solutions numerically, but we find it convenient when looking for analytical solutions.

Under this reparameterization, $\Phi'(\chi)$ simplifies to

$$\Phi'(\chi) = \frac{u'(\chi)}{2}\left[-\frac{\mathcal{R}'(u)}{\sqrt{1-\mathcal{R}'(u)^2}} + \frac{\sqrt{1-\mathcal{R}'(u)^2}}{\mathcal{R}(u)}\right]. \tag{18}$$

And the problem of finding a robust unitary reduces to picking a function $\mathcal{R}(u)$ which satisfies the following ordinary relations:

$$\mathcal{R}'(u=0) = \pm 1, \qquad \mathcal{R}'(u) \in [-1, 1] \tag{19}$$

by construction, and

$$\mathcal{R}'(u=u_f) = \pm 1, \quad \mathcal{R}(u=u_f) = \pm\sin(4\chi_f) \tag{20}$$

for robustness, and finally Eq. 9, or equivalently

$$\tan^2\phi = \lim_{\chi\to\chi_f} -16\sin^6(2\chi)\mathrm{sgn}[\mathcal{R}'(u)]R'''(u), \tag{21}$$

for targeting the unitary

$$U(\theta,\phi) = e^{-i\frac{\theta}{2}(\cos\phi\sigma_x+\sin\phi\sigma_y)} \tag{22}$$

where $\theta = 4\chi_f$. The first boundary condition above, $\mathcal{R}'(u=0)$, is required to ensure $\Phi'(\chi \to 0)$ (hence the driving field) remains finite, since the denominator $\mathcal{R}(u)$ in Eq. (18) vanishes, and the remaining two follow from robustness requirement.

Finally, $\Omega(\chi)$ is an odd function when $\mathcal{R}(u)$ is also odd, which ensures that the second of the robustness conditions in Eq. 8 is satisfied when pulsing in a symmetric time interval.

A. Examples

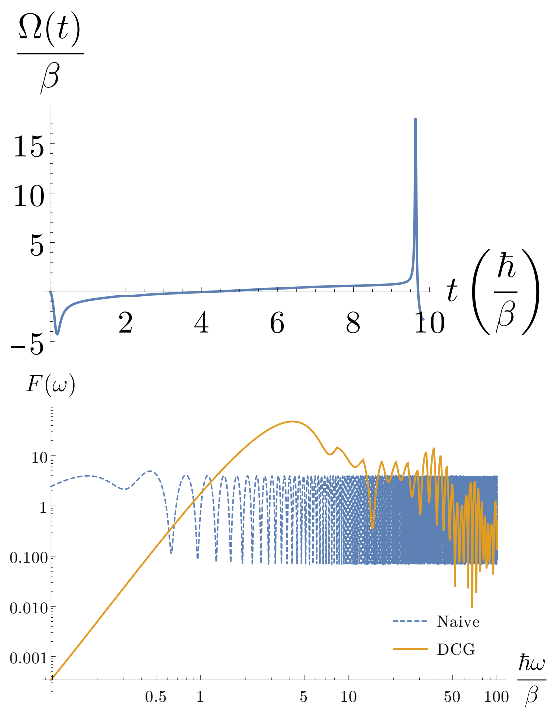

FIG. 1. (Color online) (a) Pulse shape $\Omega(\chi(t))$ (in units of β) which implements a $\theta = -\pi/2$ rotation around the axis $\boldsymbol{n} = (\cos\phi, \sin\phi, 0)$ with $\phi = \pi/9$. (b) Comparison of the leading order filter functions for the robust gate against a naive implementation using $U_{\mathrm{naive}}(t)$.

As an example, consider the following even function:

$$\mathcal{R}'(u) = a_0 + a_1\cos\left(\frac{2\pi u}{u_f}\right) + a_2\cos\left(\frac{4\pi u}{u_f}\right). \tag{23}$$

which is trivially integrable, and $\mathcal{R}(u_f)$ is simply given by $a_0 u_f$ since oscillatory functions integrate to zero at

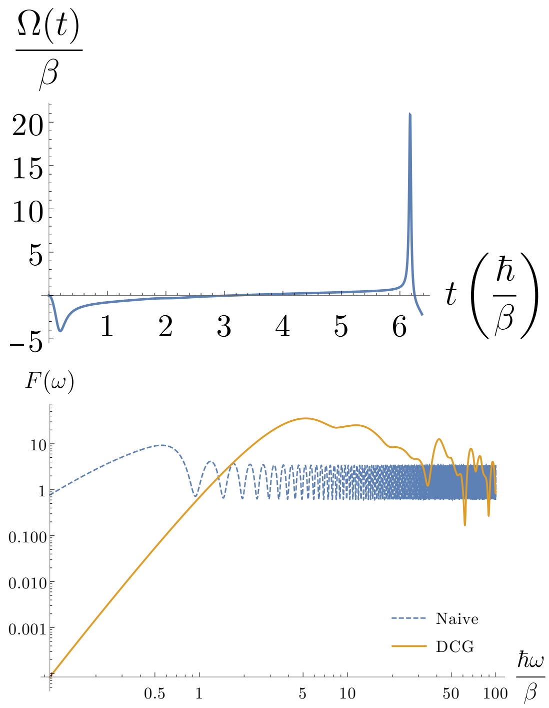

FIG. 2. (Color online) (a) Pulse shape $\Omega(\chi)$ (in units of β) which implements a $\theta = -\pi/4$ rotation around the axis $\boldsymbol{n} = (\cos\phi, \sin\phi, 0)$ with $\phi = \pi/4$. (b) Comparison of the leading order filter functions for the robust gate against a naive implementation using $U_{\mathrm{naive}}(t)$.

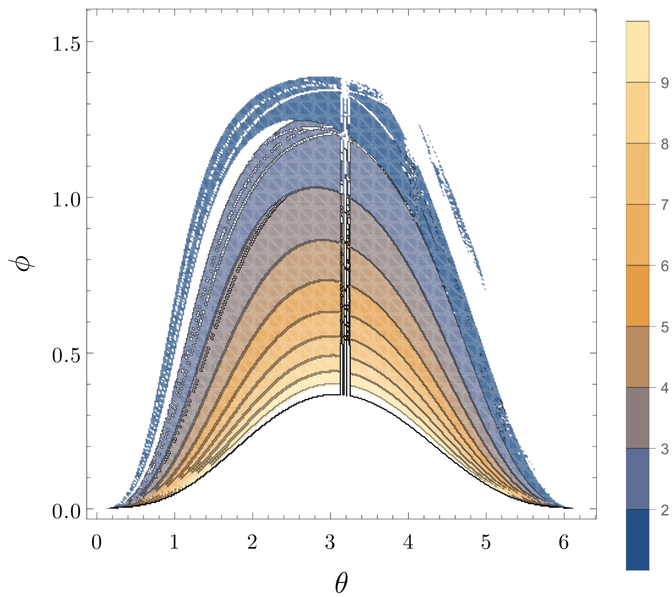

FIG. 3. (Color online) Accessible unitaries $U(\theta+2\pi n, \phi)$ when using the ansatz from Eq. 23. The minimum number of additional windings to target the unitary, n, is color coded, up to $n = 10$. White regions either require $n > 10$ or cannot be implement with this ansatz.

the final time. And by letting $a_0 = -\sin(4\chi_f)/u_f$ and $a_2 = 1 - a_0 - a_1$, we meet all the robustness conditions in Eq. 20.

We can target, say, a $\theta = 4\chi_f = 2\pi - \pi/2$ rotation around the axis given by $\phi = \pi/9$ using Eq. 21, which corresponds to the choice $a_1 \approx 0.3244$. From Eq. 6, we find the total gate time is $t_f \approx 9.84\hbar/\beta$. The resulting pulse shape is shown in Fig. 1. Similarly, for $\theta = 2\pi - 3\pi/4$, $\phi = \pi/4$, we find $a_1 \approx 0.4767$ and obtain $t_f \approx 6.38\hbar/\beta$ (Fig. 2). When using this ansatz, targeting other unitaries may require additional 2π windings in θ. The minimum number of additional windings required for targeting an arbitrary unitary $U(\theta, \phi)$ is shown in Fig. 3. Overall, these pulses require a bandwidth of $\sim 100\beta/\hbar$ when targeting fidelities above 99.99%.

It is possible to impose additional constraints, such as $\Omega(t_f) = 0$ to soften the tail, using this form of ansatz, although this requires adding higher harmonics with free coefficients of the form $a_m \cos(2m\pi u/u_f)$ leading to sharper peaks in the pulse shape.

B. Filter function

The smooth pulse is designed to cancel quasistatic noise, i.e., noise that is constant during the gate duration $t \in [-t_f, t_f]$. In practice, the noise strength may also drift during the pulse. For instance, in the context of the double quantum dot setup in [16], $\Omega(t)$ corresponds to the ESR driving amplitude and β error corresponds to exchange error induced by charge noise, which typically has a $1/f$ power spectral density (PSD). When the noise is sufficiently weak such that the error Hamiltonian $H_\epsilon(t)$ satisfies $|| \int_{t_0}^{t_f} dt H_\epsilon(t)|| \ll 1$, the average susceptibility of a quantum gate to time-dependent noise can be characterized in a perturbative manner. In this approach, the leading order error in noise-averaged fidelity is given by

$$\mathcal{F} \approx 1 - \frac{1}{\hbar^2}\sum_{i,j=1}^{3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}S_{ij}(\omega)\frac{F_{ij}(\omega)}{\omega^2}. \tag{24}$$

where $S(\omega)$ and $F(\omega)$ respectively characterize the noise and control, and are related to the error and control Hamiltonians ($H_\epsilon = \delta\beta\sigma_z$ and $H_c = \Omega(t)\sigma_x + \beta\sigma_z$ in our case) as follows. The filter function, $F(\omega)$ is given by

$$F(\omega) = [R(\omega)R^\dagger(\omega)]^T \tag{25}$$

where $R_{ik}(\omega) \equiv -i\omega\int_{t_0}^{t_f} dt R_{ik}(t)e^{i\omega t}$ and $R(t) = \mathrm{Ad}(U(t;t0)) = \mathrm{tr}(\sigma_i U(t;t_0)\sigma_j U^\dagger(t;t_0))/2$ is the adjoint representation of the time-evolution operator [16, 22]. $S_{ij}(\omega)$ is the power spectral density given by Fourier transforming the correlation between the coefficients of σ_i and σ_j terms in the noise Hamiltonian H_ϵ. In our particular case, only $S_{zz}(\omega)$ is non-zero and is given by the Fourier transform of the autocorrelation function $C_\beta(t) = \langle\delta\beta(t)\delta\beta(0)\rangle$.

We have numerically evaluated the filter functions corresponding to the gates obtained by the control pulses given in Figs.1 and 2 in a symmetric time interval from $-t_f$ to t_f. We compare their filter function to that of a naive pulse

$$U_{\text{naive}}(t) = \exp\left[-i\theta\frac{t+t_f}{2t_f}(\cos\phi\sigma_x + \sin\phi\sigma_y)\right] \tag{26}$$

which also implements the same unitary in the same amount of time. The results are shown in Figs.1 and 2. The robust gates suppress the low frequency noise much better than the corresponding naive gates, although they are more susceptible to noise at frequencies on the order of inverse gate time, $\omega \sim 1/t_f$. Thus the dynamically corrected gates (DCGs) tend to lead to higher fidelities when the noise power is concentrated at frequencies lower than $\omega \sim 1/t_f$.

IV. ROBUST PULSE SHAPES AS SOLUTIONS OF COUPLED ODE SYSTEMS

In this section, we show that the problem of finding robust pulse shapes can be converted into a set of coupled ODEs. This allows finding more general solutions, which are not restricted to antisymmetric pulse shapes, by using standard ODE solvers in a straightforward manner. This method still avoids any search over parameters, and yields solutions very quickly.

We first make a change of variables to ensure that denominator in Eq. (18) never vanishes for $\chi > 0$. A straightforward way of achieving this would be to ensure that the integrand of the denominator is always positive (or negative), which can be achieved by defining yet another function, $\gamma(\chi)$, such that

$$A\tanh(\gamma(\chi)) \equiv \alpha(\chi), \qquad \pi/2 \geq A > 0. \tag{27}$$

In terms of $\gamma(\chi)$, the robustness conditions then become

$$\gamma(0) = 0, \quad \gamma(\chi_f) = \tanh^{-1}(\alpha(\chi_f)/A), \quad \gamma'(0) = 0 \tag{28}$$

and we can solve for the second condition in Eq. (17) by considering a differential equation

$$G'(\chi) = \cos(A\tanh(\gamma(\chi)))\sin^2(2\chi), \tag{29}$$

subject to boundary condition

$$G(\chi_f) = -\frac{1}{8}\cos(\alpha(\chi_f))\sin(4\chi_f). \tag{30}$$

This function must also satisfy

$$G(0) = 0 \tag{31}$$

since $R(0) = 0$.

The rotation axis defined by the angle ϕ can be imposed via a boundary condition on $\alpha(\chi_f)$, using Eqns. (9) (15) and 17, which gives:

$$\alpha'(\chi_f) = 2\tan(\phi)/\sin(2\chi_f). \tag{32}$$

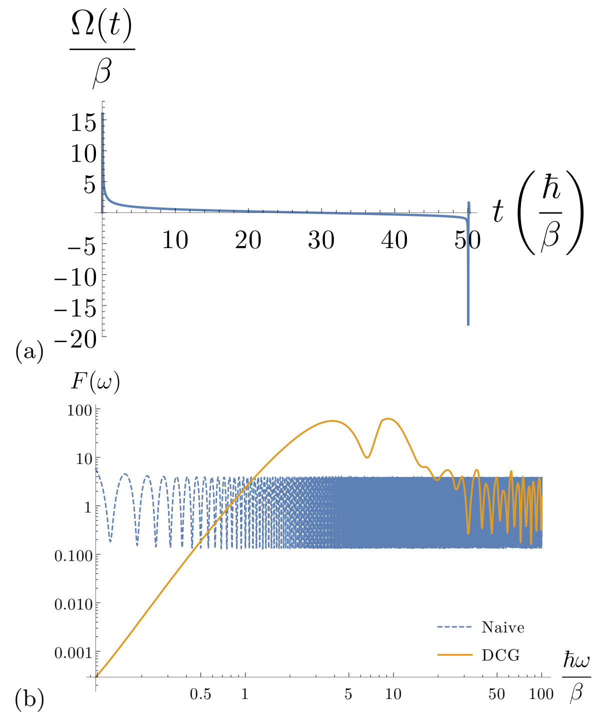

FIG. 4. (Color online) (a) The robust pulse shape $\Omega(\chi)$ (in units of β), which implements a $\theta = 9\pi/5$ rotation around the axis $\boldsymbol{n} = (\cos\phi, \sin\phi, 0)$ with $\phi = -\pi/5$. (b) Comparison of the leading order filter functions for the robust gate against a naive implementation using $U_{\text{naive}}(t)$.

We can also impose the condition that the pulse $\Omega(\chi)$ should vanish at the end by imposing a boundary condition on $\alpha''(\chi_f)$. We can see that by explicitly writing out this condition $\Omega(\chi_f) = 0$ using Eq. (5):

$$\Phi''(\chi_f) = -4\Phi'(\chi_f)\cot(2\chi_f) - [\Phi'(\chi_f)]^3\sin(4\chi_f). \tag{33}$$

Since $\Phi'(\chi_f) = \alpha'(\chi_f)/2$ and $\Phi''(\chi_f) = [\alpha''(\chi_f) - 4\alpha'(\chi_f)\tan(2\chi_f)]/2$, this can be seen as the defining condition on $\alpha''(\chi_f)$.

These two boundary conditions on the first and second derivatives of $\alpha(\chi_f)$ can readily written as corresponding boundary conditions on the derivatives of $\gamma(\chi)$, as

$$\begin{aligned}\gamma'(\chi_f) &= \frac{\alpha'(\chi_f)}{A\left(1-\frac{\alpha^2(\chi_f)}{A^2}\right)}\\ \gamma''(\chi_f) &= 2\frac{\alpha(\chi_f)\alpha'(\chi_f)^2}{A^3\left(1-\frac{\alpha^2(\chi_f)}{A^2}\right)^2} + \frac{\alpha''(\chi_f)}{A\left(1-\frac{\alpha^2(\chi_f)}{A^2}\right)}\end{aligned} \tag{34}$$

As an example, we solve for the robust pulse implementing a $U(\theta = 9\pi/5, \phi = -\pi/5)$ using the auxiliary equation

$$c\partial_t^3\gamma(t) + \partial_t^6\gamma(t) = 0 \tag{35}$$

with the choice $c = 300$, $A = \pi/2$ and $n = 0$. The pulse shape and the corresponding filter function are shown in

Fig. 4. Compared to the analytical pulse shapes based on the ansatz Eq. 23, we note that this particular auxiliary differential equation leads to numerical solutions which are sharper and take longer time to perform. However, the advantage of the numerical solutions are that they are more flexible in terms of ansatz and allow targeting arbitrary unitaries.

V. CONCLUSION

We have shown that it is possible to obtain robust quantum gates using smooth pulses in a completely analytical fashion, which only requires finding a bounded function satisfying certain local boundary conditions, at initial and final times. This eliminates non-local conditions which necessitate a numerical search over auxiliary parameters [18, 19]. Furthermore, we have shown that the problem can also be converted to a set of coupled ODEs, which further eliminates the search for such a bounded function and yields solutions very quickly using standard numerical ODE solvers. Although the presented pulse shapes tend to have narrow peaks, this is due to the simple choices of ansatz and not a fundamental limitation of our approach. Although our work assumes an SU(2) algebra, we again emphasize that our results can be applicable to two-qubit scenarios which exhibit that structure. For instance, in ^{28}Si quantum double dots [16] or superconducting qubits [21] with an always-on coupling, the Hamiltonian decouples into two SU(2) problems, and when the qubits can be addressed separately, each SU(2) subspace can be controlled separately. Our robust smooth pulses can then be used to suppress exchange noise and eliminate crosstalk while targeting a desired two-qubit unitary.

ACKNOWLEDGMENTS

UG acknowledges helpful discussions with Ed Barnes and Sophia Economou. This research was sponsored by the Army Research Office (ARO) and was accomplished under Grant Number W911NF-17-1-0287.

[1] J. T. Merrill and K. R. Brown, "Progress in compensating pulse sequences for quantum computation," in *Quantum Information and Computation for Chemistry* (2014) Chap. 10, pp. 241–294.
[2] E. L. Hahn, Phys. Rev. **80**, 580 (1950).
[3] H. Y. Carr and E. M. Purcell, Phys. Rev. **94**, 630 (1954).
[4] S. Meiboom and D. Gill, Rev. Sci. Instrum. **29**, 688 (1958).
[5] M. Veldhorst, J. C. C. Hwang, C. H. Yang, A. W. Leenstra, B. de Ronde, J. P. Dehollain, J. T. Muhonen, F. E. Hudson, K. M. Itoh, A. Morello, and A. S. Dzurak, Nat. Nanotechnol. **9**, 981 (2014).
[6] M. Veldhorst, C. H. Yang, J. C. C. Hwang, W. Huang, J. P. Dehollain, J. T. Muhonen, S. Simmons, A. Laucht, F. E. Hudson, K. M. Itoh, A. Morello, and A. S. Dzurak, Nature **526**, 410 (2015).
[7] D. M. Zajac, A. J. Sigillito, M. Russ, F. Borjans, J. M. Taylor, G. Burkard, and J. R. Petta, Science **359**, 439 (2018).
[8] T. F. Watson, S. G. J. Philips, E. Kawakami, D. R. Ward, P. Scarlino, M. Veldhorst, D. E. Savage, M. G. Lagally, M. Friesen, S. N. Coppersmith, M. A. Eriksson, and L. M. K. Vandersypen, Nature **555**, 633 (2018).
[9] C. H. Yang, K. W. Chan, R. Harper, W. Huang, T. Evans, J. C. C. Hwang, B. Hensen, A. Laucht, T. Tanttu, F. E. Hudson, S. T. Flammia, K. M. Itoh, A. Morello, S. D. Bartlett, and A. S. Dzurak, Nat. Electron. **2**, 151 (2019).
[10] R. Hanson, L. P. Kouwenhoven, J. R. Petta, S. Tarucha, and L. M. K. Vandersypen, Rev. Mod. Phys. **79**, 1217 (2007).
[11] F. A. Zwanenburg, A. S. Dzurak, A. Morello, M. Y. Simmons, L. C. L. Hollenberg, G. Klimeck, S. Rogge, S. N. Coppersmith, and M. A. Eriksson, Rev. Mod. Phys. **85**, 961 (2013).
[12] M. Bando, T. Ichikawa, Y. Kondo, and M. Nakahara, J. Phys. Soc. Japan **82**, 014004 (2013).
[13] J. P. Kestner, X. Wang, L. S. Bishop, E. Barnes, and S. Das Sarma, Phys. Rev. Lett. **110**, 140502 (2013).
[14] X. Wang, L. S. Bishop, E. Barnes, J. P. Kestner, and S. D. Sarma, Phys. Rev. A **89**, 022310 (2014).
[15] D. Buterakos, R. E. Throckmorton, and S. Das Sarma, Phys. Rev. B **97**, 045431 (2018).
[16] U. Güngördü and J. P. Kestner, Phys. Rev. B **98**, 165301 (2018).
[17] L. P. Pryadko and P. Sengupta, Phys. Rev. A **78**, 032336 (2008).
[18] E. Barnes, X. Wang, and S. Das Sarma, Sci. Rep. **5**, 12685 (2015).
[19] J. Zeng, C. H. Yang, A. S. Dzurak, and E. Barnes, Phys. Rev. A **99**, 052321 (2019).
[20] R. E. Throckmorton and S. Das Sarma, Phys. Rev. B **99**, 045422 (2019).
[21] J. M. Chow, A. D. Córcoles, J. M. Gambetta, C. Rigetti, B. R. Johnson, J. A. Smolin, J. R. Rozen, G. A. Keefe, M. B. Rothwell, M. B. Ketchen, and M. Steffen, Phys. Rev. Lett. **107**, 080502 (2011).
[22] T. J. Green, J. Sastrawan, H. Uys, and M. J. Biercuk, New J. Phys. **15**, 095004 (2013).