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# Temporal Binding of a Coherent Spectrally Translated Pulse from a Dissipative Kerr Soliton in a Synthetic Frequency Lattice

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**Abstract:** We present a dissipative Kerr soliton in a synthetic frequency lattice mediated by four-wave mixing Bragg scattering. The dual pumping creates a potential that temporally binds a coherent spectrally translated pulse to the original soliton. © 2022 The Author(s)  
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Dissipative Kerr soliton (DKS) states arise from continuous wave driving of a  $\chi^{(3)}$  dissipative system and can often be described through the Lugiato-Lefever equation (LLE) [1]. Although DKS operation has been experimentally demonstrated in various materials and spectral windows, the underlying mechanics has largely remained the same since its first demonstration [2]: a double balance of the Kerr nonlinearity with anomalous dispersion and constant driving with dissipation. Therefore, the material dispersion – especially towards the visible – presents a steep hurdle to overcome for on-demand control and generation of DKS in any spectral window of interest. Recently, the addition of a second driving field has been demonstrated for spectral extension of DKS microcombs through binding of continuous waves to the DKS [3–5]. However, questions about the physical mechanism remain, and prevent a full understanding of the range of physical phenomena possible, such as the possibility of spectrally translating a pulse.

Here, we introduce a coupled wave model that captures the nature of the coherent spectral translation, resulting in coupled Gross-Pitaevskii equations (GPE) where the presence of a cross-potential term results in a synthetic dimer system that enables binding of the two waves, although they experience different linear group velocity. This potential, driven by four-wave mixing Bragg scattering (FWM-BS) mediated by the two pumps, also exhibits phase and frequency behavior that we observe experimentally: spectral translation matching the integrated dispersion at the secondary pump frequency and translation of the DKS fixed spectral grid. Experimentally, we reconstruct this potential in the frequency and mode number space. Finally, we demonstrate that by careful choice of the secondary pumped azimuthal mode and its detuning from the drive laser, one can allow for parallel coherent spectral translation, ultimately recreating the DKS temporal pulse in a different spectral region, either close or far from the secondary pump.

FWM-BS, which directly converts a signal photon into an idler one, can also be seen as a direct coupling between modes that match the mode separation  $\Lambda$  of the two driving fields [fig. 1(a)]. This coupling between modes at different frequencies forms a synthetic frequency lattice of period  $\Lambda$  [fig. 1(b)] [6]. Therefore, it is interesting to rewrite the coupled mode equations in this frequency lattice in terms of coupling between two waves with each driven by a pump, here called  $pp$  and  $sp$ , referring to the primary pump and secondary pump respectively. From this starting point, one could recall that such a frequency lattice yields a cosine modulation in the propagation direction [7], and by accounting for the self-phase modulation and dispersion, this results in the following equations:

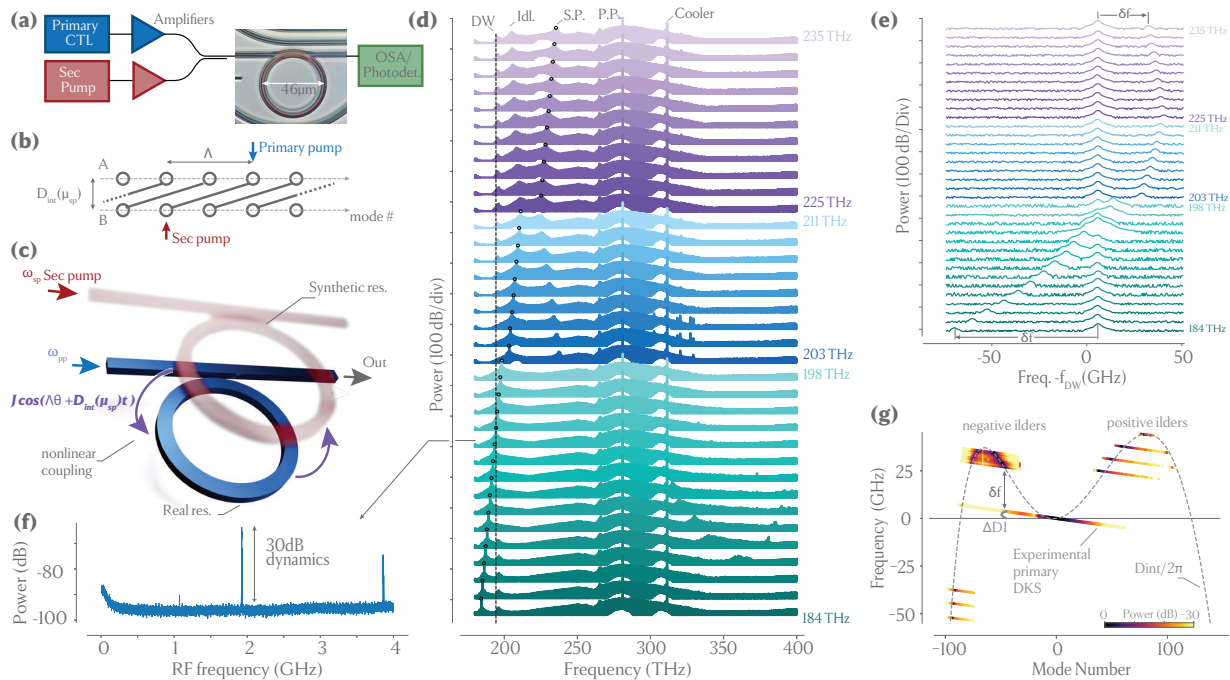


Fig. 1. (a) Dual pump driving of a microring resonator for DKS spectral translation, creating (b) a synthetic frequency lattice which in the context of DKS can be seen as a (c) synthetic dimer coupled through FWM-BS. (d) Experimental identical primary pumped DKS under different secondary pumped azimuthal modes (black dot). (e) Frequency grid offset between the primary and spectrally translated portion of the combs, following the expected trend set by the integrated dispersion. (f) RF spectrum of secondary pumping at the DW, highlighting the temporal binding of the two waves through their same repetition rate. (g) Projection of the spectral power density of the translated comb portions at their corresponding frequency grid offset. This follows the integrated dispersion closely, experimentally demonstrating the potential cross-potential term in eq. (1)

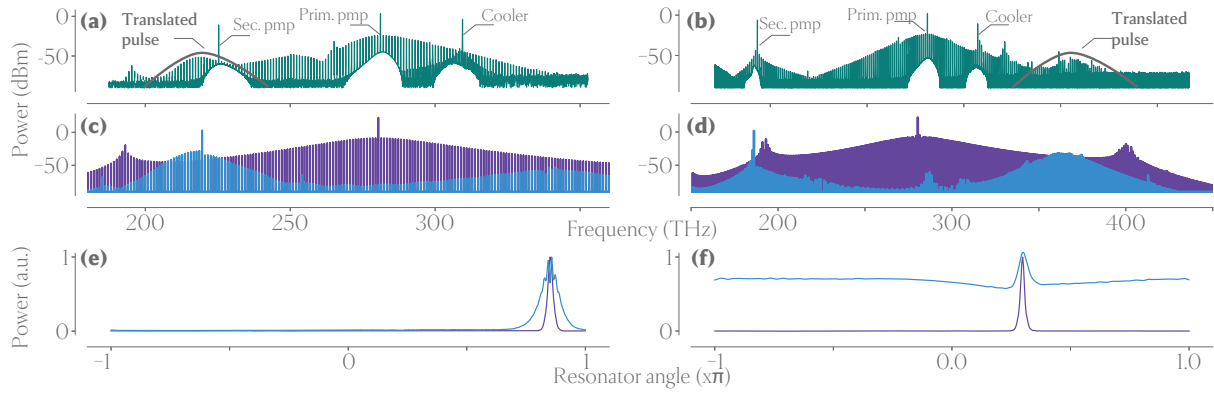


Fig. 2. (a)-(b) Experimental demonstration of pulse spectral translation using the negative and positive idler of the FWM-BS process, respectively. (c)-(d) Predicted comb spectra from the coupled-GPES simulation, with blue and purple indicating the two coupled waves. (e)-(f) Temporal envelope from the simulation exhibiting both the binding and the pulse-like nature of the spectrally translated wave.

$$\partial_t A = (\kappa/2 + i\mathcal{D})A + ig_0|A|^2A + iJ_{BS} \cos(\Lambda\theta + \Omega t)B + \sqrt{\kappa_{ext}}A_{pp}e^{i\delta\omega_{pp}t} \quad (1a)$$

$$\partial_t B = (\kappa/2 + i\mathcal{D} - i\Omega)B + ig_0|B|^2B + iJ_{BS} \cos(\Lambda\theta + \Omega t)A + \sqrt{\kappa_{ext}}B_{sp}e^{i\delta\omega_{sp}t} \quad (1b)$$

with  $A$  and  $B$  the two coupled wave fields,  $\mathcal{D}$  the dispersion,  $\Omega = D_{int}(\mu_{sp}) + \delta\omega_{sp}$  the integrated dispersion value at the secondary pump accounting for its detuning,  $\kappa$  the total losses and  $J_{BS} = g_0|AB|$  the nonlinear coherent coupling term. Interestingly, the coupling term is dependent on the resonator angle  $\theta$ , which effectively creates a potential binding both waves together. It also presents a time dependence, highlighting the shift of the spectral grid of the spectrally translated wave. This cross-potential  $J_{BS} \cos(\Lambda\theta + \Omega t)$  is coherent with the nonlinear coupling term  $ig_0B^2A^*$  and  $ig_0A^2B^*$  for eq. (1)(a) and (b) respectively, where here the phase has been explicitly written. The cross-potential is also consistent with the picture of FWM-BS arising from an effective grating moving with the speed of light in the material and beating at the frequency difference between the two driving forces ( $\Lambda$ ). This also allows for a direct comparison with other well-known systems, as such equations are equivalent to identical coupled rings constituting a photonic dimer [8]. In our case, the dimer is not due to physically distinct rings and is hence a synthetic photonics dimer [fig. 1(c)]. Although similar temporal binding has been theoretically considered in the context of propagating waveguide solitons [9], and between a DKS and its evanescent tail [10], recasting the LLE into a GPE by explicitly considering the difference of phase in the two waves exhibiting the cross potential highlights the strong temporal binding of two waves through FWM-BS.

We demonstrate such dependence on  $\theta$  (conversely mode number) and time (conversely DKS frequency grid spectral shift) by measuring a 23  $\mu\text{m}$  radius silicon nitride microring embedded in silica, with a fixed primary DKS pumped at 283 THz and a secondary pump that is tuned from one azimuthal mode to another from 184 THz to 235 THz [fig. 1(d)]. Spectrally translated teeth through FWM-BS can be observed, either on the low frequency side (negative idler - blue and purple spectra) or high frequency side of the primary pump (positive idler, teal spectra). In addition, zooming in on the dispersive wave of the primary DKS around 193 THz [fig. 1(e)], one can observe the change of the frequency grid shift due to the FWM-BS. When this shift is small enough, it can be electronically detected [fig. 1(f)], where no additional beat notes other than the harmonics are visible, reflecting the equal repetition rate in the two portions of the comb and strong temporal binding through nonlinear effect between the two waves, despite their different linear group velocity. Using the envelope of the spectrally translated portion, and the frequency grid shift measured, we show its spectral density shifted from the primary DKS [fig. 1(g)]. Overlaying the measured integrated dispersion of the resonator, it is clear that spectral translation (i.e., the presence of new idlers) occurs when  $D_{int}(\mu_{idl}^\pm) = \mp\Omega - \Delta D_1\mu$ , as expected for FMW-BS process, with  $\Delta D_1\mu$  being the difference between linear and nonlinear repetition rate. These demonstrations addressing both the azimuthal and temporal behavior of the cross-potential in eq. (1) provide experimental verification of the theoretical predictions, and confirm the coherent nature of the spectral translation of the DKS.

Using the coherent property of the spectral translation, we also show that the reconstruction of a pulse-like system at another spectral window is possible under the correct conditions. To yield such behavior, many idlers need to respect the frequency condition stated above, so that  $\partial D_{int}(\mu_{idl})/\partial\mu = 0$ . Here it is import to note this condition does not involve the secondary pump. Therefore, either using the detuning of the secondary pump to tune the idler mode [5] or using the positive BS idlers could match such conditions. Although the two driving fields experience different linear group velocity, temporal binding (i.e. forcing the same repetition rate) is nonlinearly driven. We show such behavior experimentally by secondary pumping the system at 225 THz with a detuning of  $\delta\omega_{sp} = +2$  GHz, yielding a close band of negative idlers [fig. 2(a)], and at 187 THz with  $\delta\omega_{sp} = -2$  GHz, yielding a band of idlers far from the secondary pump (positive idler) [fig. 2(b)]. The spectral envelope of the translated idlers has a similar shape as the original DKS around the primary pump, and is significantly different than the Lorentzian-decay type dispersive wave shape observed for spectral translation outside of the  $\partial D_{int}(\mu_{idl})/\partial\mu = 0$  condition [3]. These observations are in good agreement with simulations [fig. 2(c)-(d)] that also capture the change of sign of the positive idler frequency grid, and therefore a fast time modulation of the translated pulse [fig. 2(e)-(f)]. Together, the experiments and simulations illustrate the likely pulsed nature and binding of the translated pulse to the primary DKS.

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